

UW Math Circle: Halloween Edition

October 30, 2014



1. The eighth graders at Haunting Middle School all dressed up for Halloween as either a bat or a cat. At lunch, they all bought either a spooky sandwich or a mysterious muffin. If all of the bats bought a sandwich and all of the cats bought a muffin, they would have spent one cent less than if all of the cats bought a sandwich and all of the bats bought a muffin. If you know that the number of bats is greater than the number of cats, find the difference between the two.



2. Amy went trick-or-treating and got some number of candy bars, all of which happened to be rectangular prisms of the same size. Amy realized that she could stack her candy bars in an $n \times n \times n$ cube, with $2n$ candy bars leftover, for some natural number n . Prove that the total number of candy bars Amy got was divisible by 3.

3. The Wicked Witch has enchanted her broom to not only fly, but also clean the floors of her lair. However, the spell went a little wrong and the broom will only sweep the floors if the number of dust particles on the floor is a multiple of 7. She lives in a massive castle and hasn't cleaned in a long time. If the current number of dust particles is exactly $2222^{5555} + 5555^{2222}$, will the broom sweep the floor?



4. Joe wants to dress up as a number for Halloween, and has bought one hundred 0 stickers, one hundred 1 stickers, and one hundred 2 stickers. He wants to use all the stickers as the digits of his number, but can arrange them in any order. He thinks this is the “perfect” Halloween costume, but can such a number be a perfect square?