

UW Math Circle

October 1, 2015

Homework

1. Kristin needs two rubber bands for her hair. If she has 5 red rubber bands, 2 blue ones, 4 purple ones, and 12 black ones, what is the smallest number of rubber bands she could grab to ensure she has two of the same color?
2. Suppose you have 9 coins, identical in appearance. You know one of them is fake and lighter than the others. If you have a balance scale, what is the smallest number of weighings it would take to find the fake coin?



3. In a faraway swamp, there are two kinds of animals: leeches and toads. If you ask a leech a question, it will always lie to you, and if you ask a toad, it will always tell the truth. There are four animals living in the swamp named Dan, David, Debbie, and Diana. They made the following statements:

Dan: "Diana and I are different species."

David: "Debbie is a leech."

Debbie: "David is a leech."

Diana: "Of the four of us, at least two are toads."

How many of these animals are leeches, and how many are toads?



4. There are 5 coins, three of which are real. One is fake and lighter than the real coins and the other is fake but heavier than the real coins. How can you determine which coins are fake using only three weighings?