

UW Math Circle: Reindeer on a Rope

December 10, 2015



Today's problem set deals with reindeer. Reindeer are hungry creatures and consume everything they can reach. Because of this, they are usually kept on a rope.

1. Draw the section of a pasture consumed by a reindeer if the reindeer is tied to a single stake planted in the pasture.
2. A mathematician took a walk on a field holding a reindeer on a 1-meter-long rope. The mathematician's path was rectangular with dimensions 3 meters by 5 meters. Draw the section of the field the reindeer will have consumed by the end of the mathematician's walk.
3. How can a reindeer be constrained to an eye-shaped field? In other words, how can the reindeer be tied using ropes and stakes, so that it can eat only the grass in the field?
4. A rope has been stretched between two stakes in a field. A reindeer is tied to this rope with another rope that is free to slide along the first rope. What is the shape of the portion of the field the reindeer can eat?
5. How can a reindeer be constrained to a field in the shape of a...
 - (a) semicircle?
 - (b) square?
 - (c) rectangle?
6. How can a reindeer be constrained to a field in the shape of a...
 - (a) triangle?
 - (b) regular hexagon?
7. Dogs can be used to herd reindeer because a reindeer will not occupy a space that a dog can reach. However, we can't let a dog run free since it will chase the reindeer constantly, and never let the reindeer rest or eat.
 - (a) How can one dog hold a reindeer in a ring?
 - (b) What about in a semicircle?
 - (c) Using dogs, contain an untied reindeer in a triangle.
8.
 - (a) The fence of the Reindeer-R-Us farm is triangular in shape. Two reindeer are tied to the fence with ropes, each at the midpoint of two different sides. The lengths of the ropes are equal to half the lengths of the section of fence they are tied to. Can the reindeer eat all of the grass inside the fence?
 - (b) What if the fence is a quadrilateral, and there are reindeer tied at the midpoints of each of the four sides with ropes that are half the length of the side they are tied to?

9. In how many different ways can you fill in the blanks with standard operations (plus, minus, times, division, exponent) and place parentheses as needed to make true statements?

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 1$$

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 2$$

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 3$$

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 4$$

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 5$$

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 6$$

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 7$$

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 8$$

$$1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 = 9$$

10. In how many ways can you fill an $m \times n$ checkerboard with 0s and 1s so that the sum of the numbers in each row and each column is even?