UW Math Circle November 12, 2015 Homework

1. Noah and his friends are playing Duck Duck Goose, and they're sitting in a big circle in such a way that both neighbors of each person are the same gender. If there are 9 girls in the circle, how many boys are playing Duck Duck Goose?



2. Take any 6 consecutive numbers, like 17, 18, 19, 20, 21, 22. Let *a* be the sum of the first 3 numbers and *b* be the sum of the last 3 numbers. Is it possible that ab = 9999999?

3. Take any 17 digit number, and reverse its digits to get another 17 digit number. Show that the sum of these two numbers must have at least one even digit.

4. We say that 2 divides an integer n if there exists another integer k so that 2k = n. For example, 2 divides -6 since 2(-3) = -6, but 2 does not divide 5.



Show that if n is an integer and 2 divides n^2 , then 2 divides n. There are lots of ways to do this; see if you can do it using mod 2 arithmetic.

5. Show that if 2 divides n and 3 divides n, then 6 divides n as well.