UW Math Circle

November 19, 2015

Compute the following quantities.

1. $13 \equiv$ _____ mod 4
2. $73 \equiv$ _____ mod 11
3. $22 \equiv$ _____ mod 9
4. $15 \equiv$ _____ mod 6
5. $32 \equiv$ _____ mod 10
6. $4 \cdot 32 \equiv$ _____ mod 10
7. $17 \equiv$ _____ mod 3
8. $19 \equiv$ _____ mod 3
9. $17 + 19 \equiv$ _____ mod 3
10. $-4 \equiv$ _____ mod 3
11. $-21 \equiv$ _____ mod 5
12. $14 \equiv$ _____ mod 3
13. $27 \equiv$ _____ mod 4
14. $11 \equiv$ _____ mod 4
15. $5 \equiv$ _____ mod 4
16. $27 \cdot 11 \equiv$ _____ mod 4
17. $27 \cdot 11 \cdot 5 \equiv$ _____ mod 4
18. $n^2 + 2 \equiv$ _____ mod n
19. $3(n + 1) \equiv$ _____ mod n
20. $(2n + 1)(n + 2) \equiv$ _____ mod n
21. $2 + 4 + 6 + \cdots + 2n \equiv$ _____ mod n
22. $1 + 2 + 3 + \cdots + n \equiv$ _____ mod n

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1. Show that $n^3 + 2n$ is divisible by 3 for any integer n.

2. Show that a number is divisible by 4 if and only if its last two digits are divisible by 4.

3. Develop a rule for deciding if a number is divisible by 7.

4. What is the last digit of 2013^{2013}? How about 2014^{2014}?

5. When Peter broke his piggy bank, it contained no more than 100 coins. He divided coins into piles of 2 coins each, but was left with one extra coin. The same happened when Peter divided the coins into piles of 3 coins, piles of 4 coins, and piles of 5 coins. Each time he was left with one extra coin. How many coins were in the piggy bank?