UW Math Circle
December 3, 2015

1. James wants to schedule an appointment for ten thousand days from today, but needs to know what day of the week that is. What day of the week will it be $10,000$ days from today?

2. What is the last digit of $1^2 + 2^2 + 3^2 + \cdots + 99^2$?

3. Show that 2015 can't be written as $2015 = a^2 + b^2$, where a and b are integers.

4. Determine a rule to test if an integer cannot be written as the squares of two other integers. Your test should tell you very quickly that 3, 107, 2015, and $1,000,000,000,003$ cannot be written as the squares of two integers.

   Can you find an example of a prime number that passes your test, but is not the sum of squares of two other integers?

5. We say that an integer m has an **inverse** mod n if there is some other integer l such that $ml \equiv 1 \bmod n$. For example, 5 has an inverse mod 9 because $5 \cdot 2 = 10 \equiv 1 \bmod 9$.

   (a) Does 5 have an inverse mod 7?
   (b) Does 5 have an inverse mod 8?
   (c) Does 5 have an inverse mod 10?

   Determine a rule that tells you if an integer m does or doesn't have an inverse mod n. (Hint 1: you might want to do a few more specific examples like parts (a), (b), (c) above. Hint 2: it might be easier to determine when an integer *doesn't* have an inverse.)