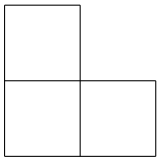
UW Math Circle

March 31, 2016

1. A tromino is an L-shaped piece, drawn below. Is it possible to cover an 8×8 chessboard with its upper left corner removed with trominoes? How about a 16×16 board with its upper left corner removed? 32×32? $2^n \times 2^n$?



2. You are playing the game of *'The Towers of Hanoi'*. It has three spindles on a base, with n rings on one of them. The rings are arranged in order of their size - from largest on the bottom to smallest on the top. It is permitted to move the highest (smallest) ring on any spindle onto another spindle, except that you cannot put a larger ring on top of a smaller one. Prove that:

   (a) It is possible to move all the rings to one of the free spindles;

   (b) You can do so using $2^n - 1$ moves.

   (c) It is not possible to do so using fewer moves.