UW Math Circle February 26, 2015 Homework

1. Show that if you have any m lines drawn in the plane, then it is possible to color the regions between the lines black and white in such a way that no two neighboring regions are the same color.

2. Prove that the number 111...111 (3^n 1's) is divisible by 3^n .

3. Tim is an astronaut and he is planning on flying his spaceship around the equator of the Moon. He sent a rover to the moon to deposit fuel at equal intervals around the equator, but there was a computer glitch and the rover dropped fuel packets randomly instead. All Tim knows is that the total amount of fuel in the packets is exactly enough for him to get around the equator once. Prove that he can find a starting point on the equator to go around it without running out of fuel.

