UW Math Circle

April 28, 2016

Today, we're going to play a game:

Pick two numbers m and n.

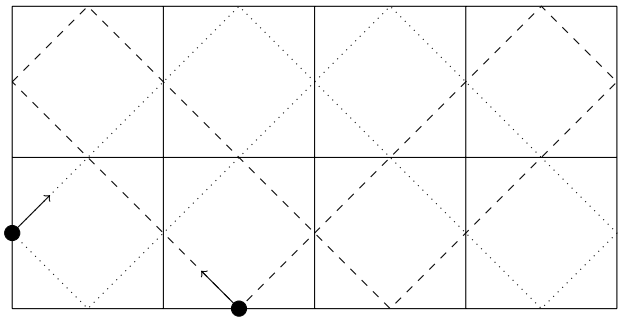
Draw a grid of squares with m rows and n columns. Now pretend there is a mirror around the outside of the rectangle.

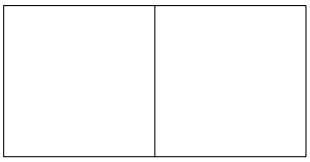
At the midpoint of any of the squares around the outside of the grid, you can place a laser(!) and shoot it either directly northeast, northwest, southwest, or southeast.

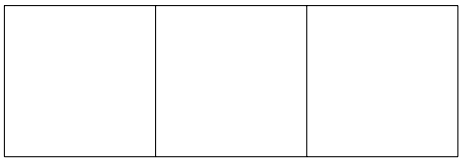
The laser beam will then reflect off the mirrors around the outside of the grid, and eventually return to where it started.

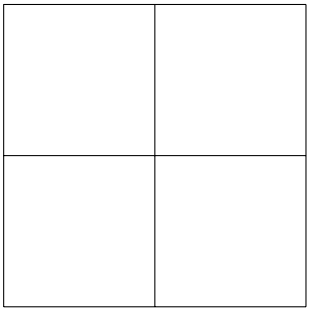
Question: How many lasers do you need to place if you want the midpoint of each of the squares around the perimeter to be hit by a laser?

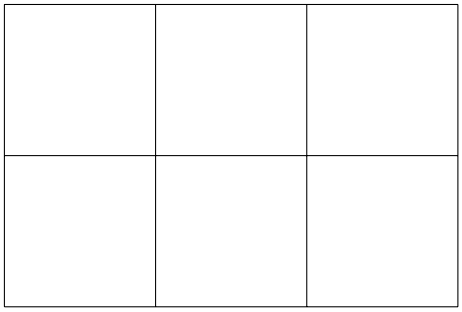
For example, if $m = 2$ and $n = 4$, you need two lasers. The grid is drawn in solid black, the first laser is a dotted line, and the second laser is a dashed line.

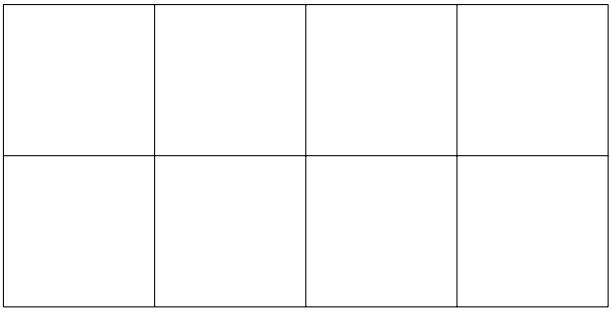


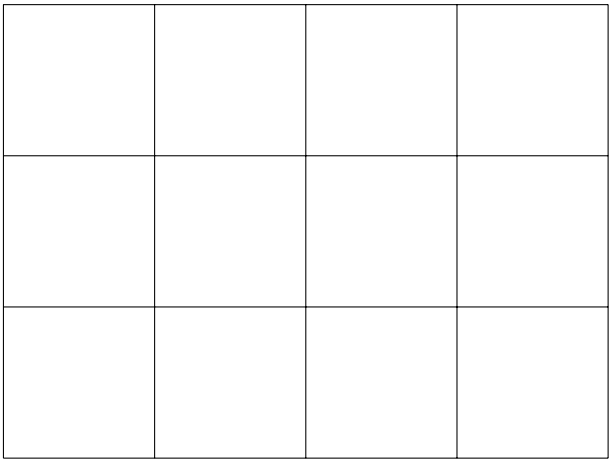


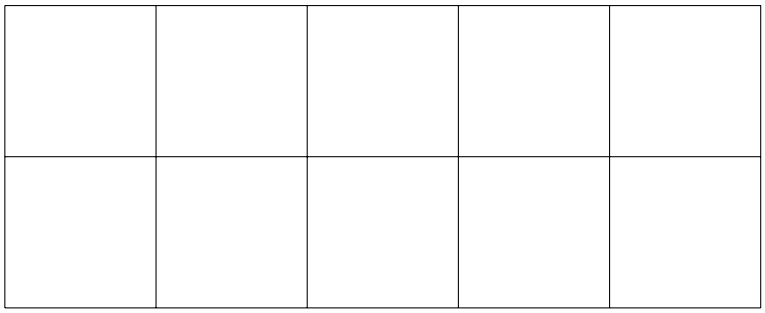


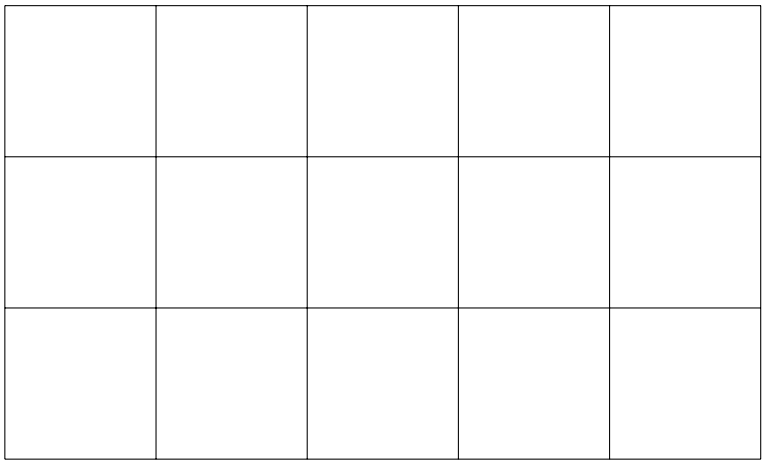


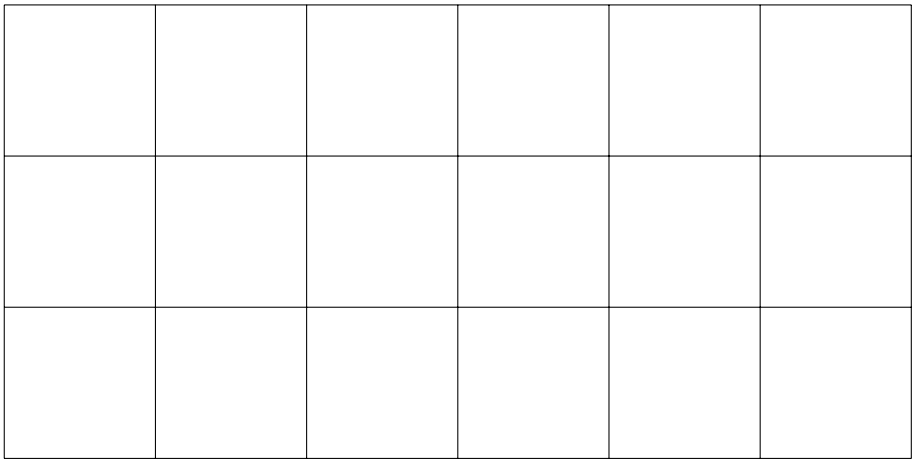


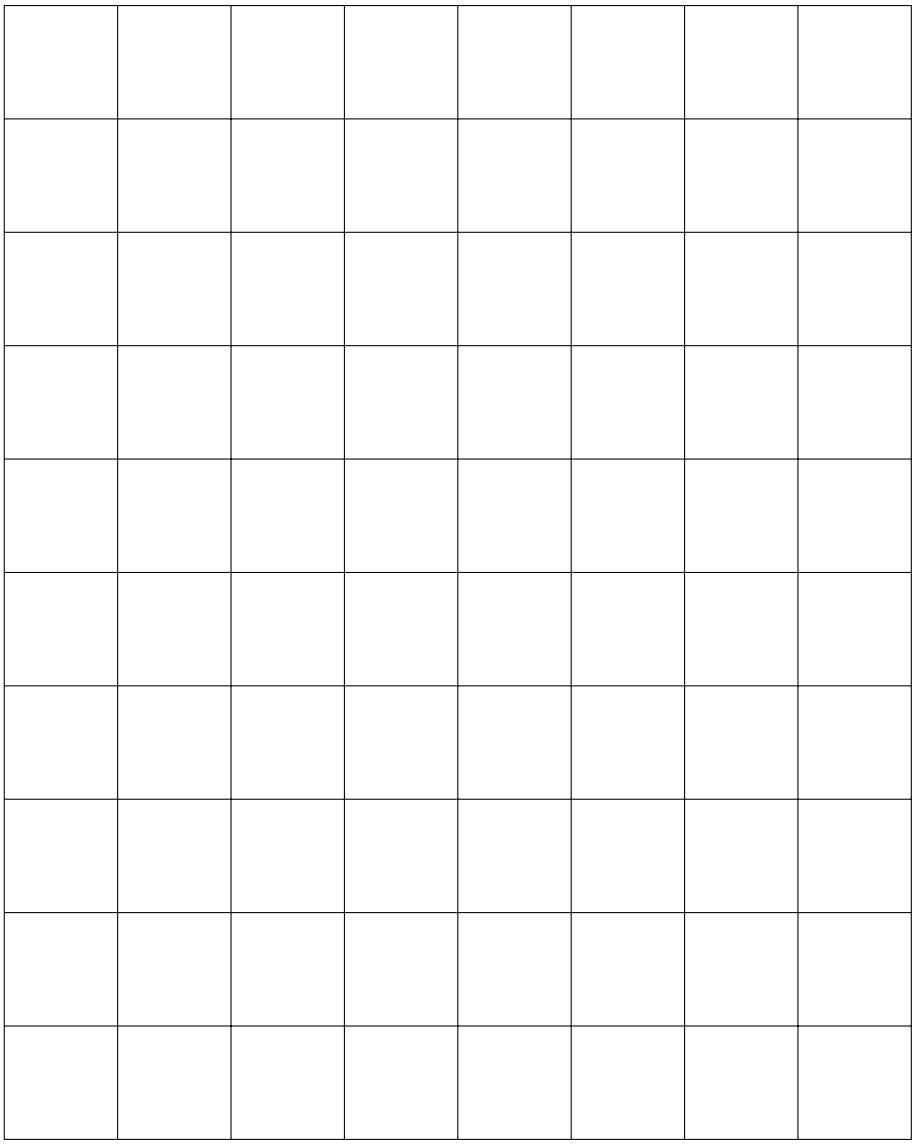


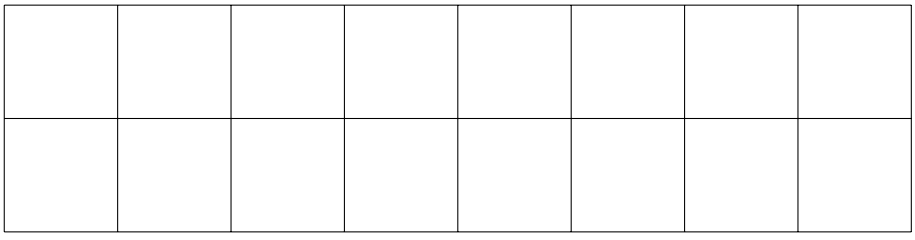












The first couple problems are from last week's homework, if you've already done them feel free to move on! You might use Fermat's little theorem.

1. $3^{31} \equiv ?\pmod 7$?

2. What is $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60}$ modulo 7?

3. What is the units digit of 2^{2008}?

4. Modulo 10, we have that $2^1 = 2$, $2^2 = 4$, $2^3 = 2$, $2^4 = 6$, $2^5 = 2$, $2^6 = 4$, ... Do you see a pattern? Show that this pattern always holds for the 10s digit of powers of 2.

5. Let $N = 2008^2 + 2^{2008}$. What is the units digit of $N^2 + 2^N$?