UW Math Circle

January 28, 2016

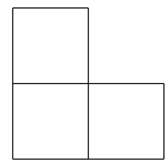
1. 100 students are sitting at a round table, and more than half of them are girls. Prove that at least two girls are seated directly across from one another.

2. Fifteen squirrels gathered a total of 100 nuts. Prove that at least two of the squirrels gathered the same number of nuts.

3. Suppose there is a 10 ft $\times$ 100 ft wall painted in three colors: red, blue, and yellow. Prove that no matter how the wall is painted, there is a rectangle with all its vertices painted the same color.

4. Prove there exist two powers of 2 that differ by a multiple of 2015.

5. (a) What is the largest number of squares on an 8×8 chessboard which can be colored orange, so that in any "tromino" (pictured below; you can rotate it any way you'd like), there is at least one square that is NOT orange?
   (b) What is the smallest number of squares that can be colored orange such that in each tromino, at least one square IS orange?



6. Prove that for any set of 17 positive integers, there is some subset of 5 of them whose sum is a multiple of 5

7. Show that if 16 people are seated in a row of 20 chairs, then some group of 4 consecutive chairs must be occupied.

8. Show that in any group of six or more people, at least three people all either know each other or are all strangers. Does this necessarily happen in any group of 5 people?