UW Math Circle

February 4th, 2016

Homework

We think of a **graph** as a set of dots (vertices) with edges between them, where each edge is attached to exactly two vertices.

1. Due to some recent galactic advances, transporters allow you to make the following jumps between planets: Earth–Mercury, Pluto–Venus, Earth–Pluto, Pluto–Mercury, Mercury–Venus, Uranus–Neptune, Neptune–Saturn, Saturn–Jupiter, Jupiter–Mars, and Mars–Uranus. Is it possible to travel from Earth to Mars?

2. In the faraway country of Septiland, there are 15 cities. Each city is connected to at least 7 other cities by a road. Show that it is possible to drive between any two cities, possibly passing through some other cities along the way.

3. (a) At a campground, there are paths between the campsites. A path goes straight from one campsite to another, there are no forks in a path (but a campsite can have multiple paths entering it). Wherever a path enters a campsite, there is a large stone. What can you say about the total number of stones?

   (b) What can you say about the number of campsites that contain an odd number of stones? (If a campsite contains an odd number of stones, there are an odd number of paths leaving it).

4. (a) We color each edge of the complete graph on 6 vertices (this is the graph with 6 vertices where every vertex is connected to every other vertex by an edge) red or blue. Show that no matter how we color this graph, there is a set of three vertices so that all the edges between them are either red or blue.

   (b) Earlier you were asked to show that at a party with 6 or more people there were three people who either all knew each other or were all strangers to one another. Explain how the previous problem solves this question.

   (c) Is the claim from part a) true if we are coloring a complete graph on 5 vertices instead?