# UW Math Circle
February 11, 2016
Homework

1. In the land of Tripleville, there are 3 roads leading in and out of each city. Is it possible for there to be 99 cities in Tripleville? What about 100 cities?

2. In the neighboring land of Centropolis, 100 roads lead out of each city, and it's possible to travel along these roads from any city to any other. One of the roads is closed for repairs. Prove that it's still possible to get from any city to any other city.

3. An airline company is planning to fly to 5 different cities, and from each city, they plan have nonstop service to 15 others (nonstop service means that if they have a direct flight from city A to city B, then there is also a direct flight from city B to city A). Prove that this flight plan is impossible.

4. If you start with four knights placed on a 3 × 3 chess board as shown on the left, is it possible to move them into the position on the right if two knights are never allowed to occupy the same square?
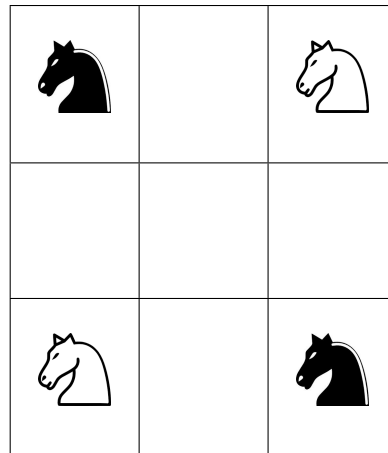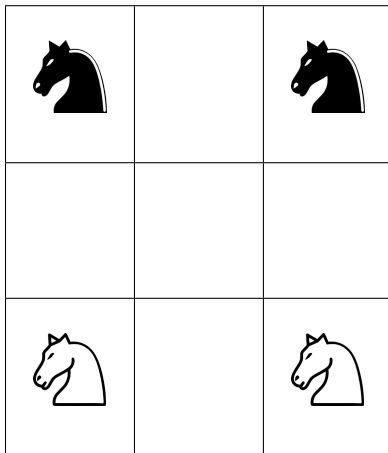
5. On the worksheet, you showed that any planar graph can be colored with six colors so that no two adjacent vertices are the same color.

    (a) Find a planar graph whose vertices cannot be colored with three colors so that no two adjacent vertices share a color.

    (b) Show that every planar graph can have its vertices colored with 5 colors so that no two adjacent vertices share a color.