

UW Math Circle  
February 18, 2016  
Homework

1. At Middleton Middle School, there are 6 clubs. The following students are members of each club:

Club	Members
Drama Club	Andrew, Bob, Catherine
Math Team	Andrew, Catherine, Dave
Recycling Club	Bob
French Club	Catherine, Dave, Elise
Spanish Club	Bob, Elise
Debate Team	Andrew, Dave

What is the minimum number of days needed for the clubs to meet, provided that no two clubs with a shared member meet on the same day? Can you interpret this problem in terms of coloring a graph?

2. Jenny works at a zoo and is in charge of building habitats for the animals. She needs to build a habitat for anteaters, brown bears, cougars, deer, elephants, flamingos, giraffes, hippopotamuses, iguanas, and jaguars. But, some of the animals are more ... predatory than others. Therefore,
- (a) Hippopotamuses cannot live with deer or giraffes.
  - (b) Deer, flamingos, and anteaters all need separate habitats.
  - (c) Iguanas can share their habitats only with jaguars.
  - (d) Bears and elephants don't get along; neither do deer and elephants.
  - (e) Jaguars scare hippopotamuses, cougars, giraffes, and brown bears.
  - (f) Deer and flamingos are also afraid of jaguars.

What is the fewest number of habitats Jenny could build? Can you interpret this problem in terms of coloring a graph?



3. We call a graph **planar** if we can draw it in the plane without any of the edges crossing. Remember that, for a planar graph,  $V - E + F = 2$  and  $2E \geq 3F$ , where  $V$ ,  $E$ , and  $F$  are the number of vertices, edges, and faces. We say a graph is **complete** if every vertex is connected to every other vertex.

- (a) Prove that, for a planar graph with at least three vertices,  $E \leq 3V - 6$ .
- (b) If  $K_n$  is the complete graph with  $n$  vertices, for what values of  $n$  is  $K_n$  planar? (Hint: can you use the coloring theorems?)
- (c) Which of the following graphs are planar? If the graph is not planar, what is the minimum number of crossings in the graph?

