Auction III

UW Math Circle

9 June 2016

1. You need to transport 3000 apples from Seattle to Las Vegas, 1000 miles away. Your truck can carry 1000 apples at a time, but you need to consume 1 apples for every mile you travel *away* from Seattle (you don't consume anything traveling *back* towards Seattle). How many apples could you bring to Las Vegas? (Note: it is permissible to drop some apples off at the side of the road, and pick them up later.)

Figure 1: Some apples.

2. An integer's abundance ratio is the ratio of the sum of its divisors to itself. For example, 36 has factors

$$1, 2, 3, 4, 6, 9, 12, 18, 36$$

The sum of the factors of 36 is 91. So the abundance ratio of 36 is $91/36 \approx 2.528$. Find a number, *not containing the digit 0*, less than 1000000, with the largest abundance ratio.

3. You have 8 pancakes in a stack, one with one blueberry, one with two blueberries, one with three blueberries, and so on. Count the number of blueberries in the top pancake and call that number N. Pick up the stack of the top N pancakes and flip it upside down. Repeat this procedure until the pancake with one blueberry ends up on the top. Find an initial arrangement of pancakes that takes the largest number of flips for the pancake with one blueberry to end up on the top.

Figure 2: Some pancakes, with no blueberries.

4. For a pair of words of the same length, we can calculate their partner score as follows. The pair gets 1 point for each letter they have in common in a different position within each word.

   For example, the pair OPENS and CLOSE has a partner score of 3, and the pair JUMP and PUMA has a partner score of 1.

   Find a pair with the largest partner score possible.

Figure 3: The words aren't even the same length.

5. Initially a "1" is written on the board. Choose two positive integers a and b that aren't 1. Every minute, every number x on the board is replaced with both $ax + a$ and $bx + b$. For example if I chose 2 and 5 I would write

$$\{1\} \to \{4, 10\} \to \{10, 25, 22, 55\} \to \cdots$$

   Notice that after 2 steps I repeated a number, the number 10. Choose a and b such that the time it takes to repeat a number is as large as possible.

6. Four positive integers less than 100 are written at the corners of a large square. Every minute, four dots are drawn at the midpoints of the square's edges, and labeled with the positive difference between the adjacent numbers. These midpoints are connected to form a new square, and then the process repeats. This continues until we form a square whose corners are all labeled 0. Find initial labels of the square that take the most steps for this to occur.

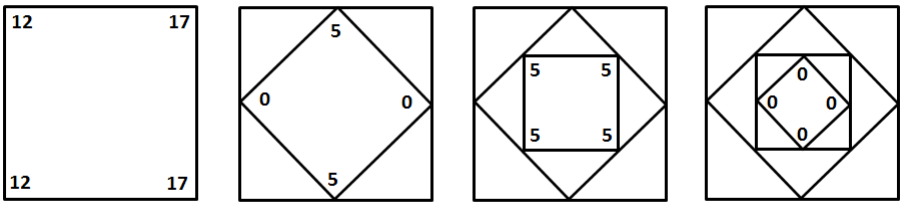

Figure 4: An example.