UW Math Circle
September 28th, 2017

Tucked away in a secret corridor of Hogwarts School of Witchcraft and Wizardry, there are a number of magical closet doors. If you open one of these closet doors, you might find Harry Potter, who will teach you everything you need to know about magic. You might also find Voldemort, who ... won't.

Because Hogwarts is a magical place, you might find Harry behind multiple doors. You might also find Voldemort behind multiple doors.

Professor Snape has put a sign on each door telling you what's inside, but not all of the signs are true!

1. When you enter the secret hall, you encounter two closet doors. "One of the signs is true," says Professor Snape, "and the other is false."

I	II
Harry Potter is behind this door. Voldemort is behind the other one.	Harry Potter is behind one of these doors. Voldemort is behind the other.

   Which door should you open?

2. When you enter the secret hall, you encounter two closet doors. "This time, either both signs are true or both signs are false," says Snape.

I	II
Harry Potter is behind at least one of these doors.	Voldemort is behind the other door.

   Which door should you open?

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1. Austin tore out a section of successive pages from a book. The first page he tore out was numbered 165, and the last page he tore out was also numbered with the digits 1, 6, and 5 in some order. How many pages did Austin tear out of the book?

2. Suppose you have 9 coins, identical in appearance. You know one of them is fake and lighter than the others. If you have a balance scale, what is the smallest number of weighings it would take to find the fake coin?

3. 5 pirates of different ages have a treasure of 100 gold coins. On their ship, they decide to split the coins using this scheme: the oldest pirate proposes how to share the coins, and ALL pirates (including the oldest) vote for or against it. If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain.

   As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if she voted for *or* against a proposal, she will vote against so that the pirate who proposed the plan will be thrown overboard.

   Assuming that all 5 pirates are intelligent, rational, greedy, and do not wish to die, what is the maximum number of coins the oldest pirate can get?

4. Using only one cut, can you cut the shape below into two pieces that can be moved to form a 4×6 rectangle? (You can only make one cut, but it can be a jagged line.)
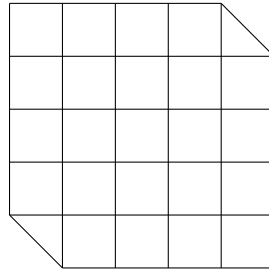
5. A squirrel is trying to run from one corner of the city to another, but can't run on the ground because she's being chased by a dog! She is running along telephone wires, which are arranged in a 6×6 grid:
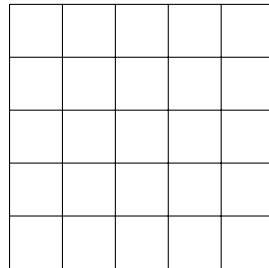
   If she starts at the bottom left corner and wants to get to the top right corner and only runs up and to the right, how many different paths can she take?

6. How many ways are there to cover a $2 \times n$ chessboard with dominoes? (Assume each domino covers exactly 2×1 or 1×2 squares.)