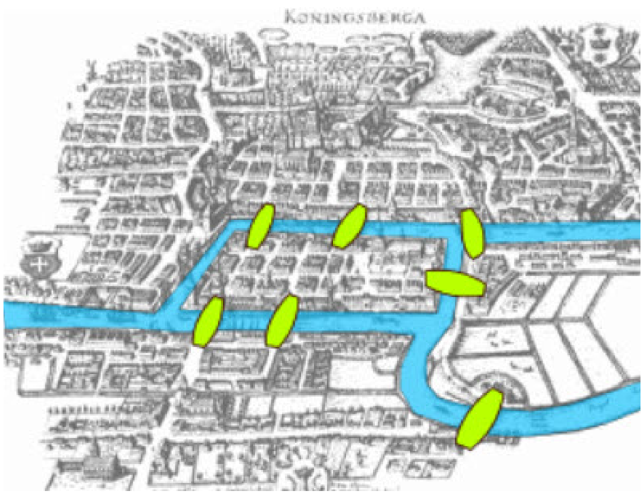
UW Math Circle

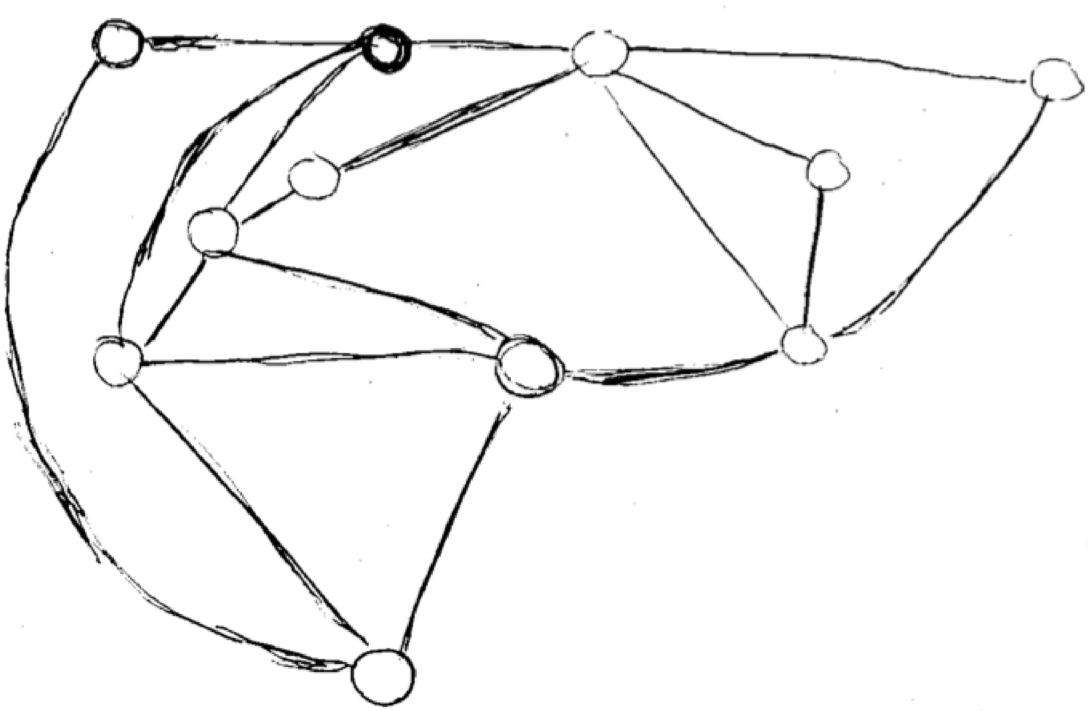
October 5th, 2017

1. The city of Königsberg has seven bridges, as pictured below. Can you go through the city of Königsberg and cross each bridge exactly once? (You're not allowed to swim, tunnel, fly, go around, etc.) Either show that you can do this, or explain why you can't.

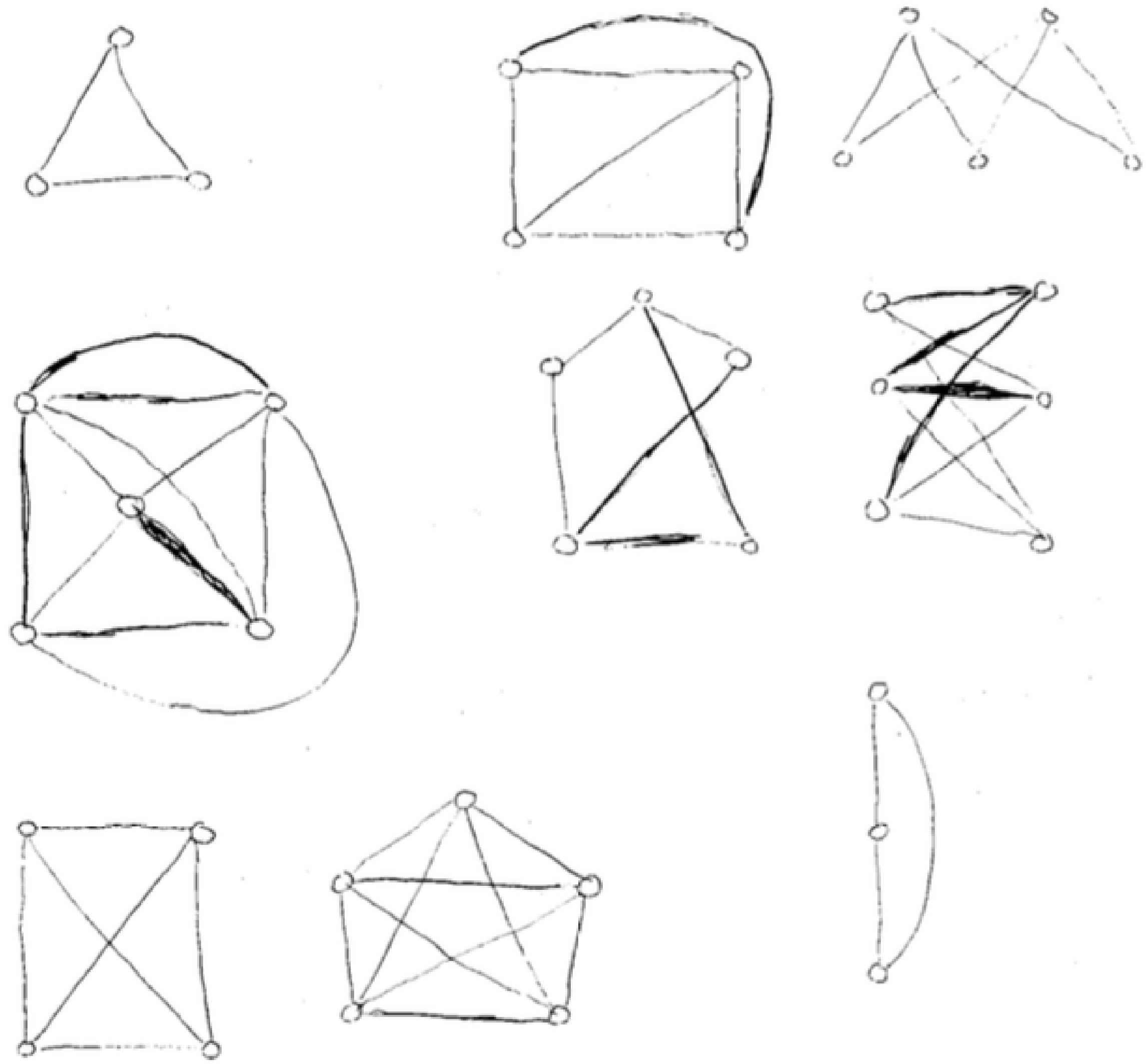
   What if you add some bridges, or take some away?

2. Below is a map of the archipelago of Euleria. Euleria has 11 islands and 18 bridges between them. Can you travel through Euleria and cross each bridge exactly once?



3. Which of these are the same, and which are different?



4. If the above pictures were maps of groups of islands, where the dots represent islands and the lines represent bridges, which ones could you travel throughout while crossing each of the bridges exactly once?

5. Given any such map, find a criteria to determine whether or not you can travel through the region, crossing each bridge exactly once.

6. We call a graph **planar** if we can draw it in the plane without any of the edges crossing. A *face* of a planar graph is a region bounded by the edges. Note that the region outside a graph is also a face.

   (a) What is the minimum number of edges bounding a face for a graph with more than three edges and without multiple edges between two vertices?

   (b) For a planar graphs without multiple edges between vertices and with more than 3 edges, show that $2\#$ of edges $\geq 3\#$ of faces.

7. Which of the following graphs are planar? For each planar drawing that you find, find $\#\text{vertices} - \#\text{edges} + \#\text{faces}$.