UW Math Circle

October 12, 2017

Homework

1. In the land of Tripleville, there are 3 roads leading in and out of each city. Is it possible for there to be 99 cities in Tripleville? What about 100 cities?

2. In the neighboring land of Centropolis, 100 roads lead out of each city, and its possible to travel along these roads from any city to any other. One of the roads is closed for repairs. Prove that its still possible to get from any city to any other city.

3. A coloring of a graph is an assignment of colors to the vertices of a graphs, where if two vertices are connected by an edge then they must be different colors. Can you find a planar graph that cannot be colored with two colors? Three colors? Four colors?

4. Find the smallest number of colors needed to color each vertex of the graph below so that no two connected vertices share an edge.