UW Math Circle

October 19, 2017

The first page is problems from last week that not everyone got to. Remember: we say a graph is *planar* if we can draw it on a sheet of paper in a way where none of the edges cross. For any graph, V is the number of vertices and E is the number of edges. If we have a planar graph and a drawing of it where none of the edges cross, F is the number of faces: the regions that the edges cut the plane into. Remember, the outside of the graph counts as a face.

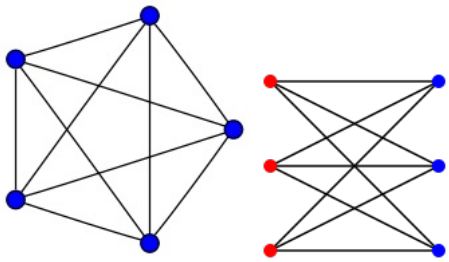
Question: *Can you draw a graph in the plane so that no edges cross in two different ways, so that F is different for the two different drawings?* You'll find out below!

1. Show that, for any planar graph with at least two edges, $2E \geq 3F$.

2. (a) Show that, for any planar graph, $V - E + F = 2$.

   (b) Does this answer the question at the beginning of the page?

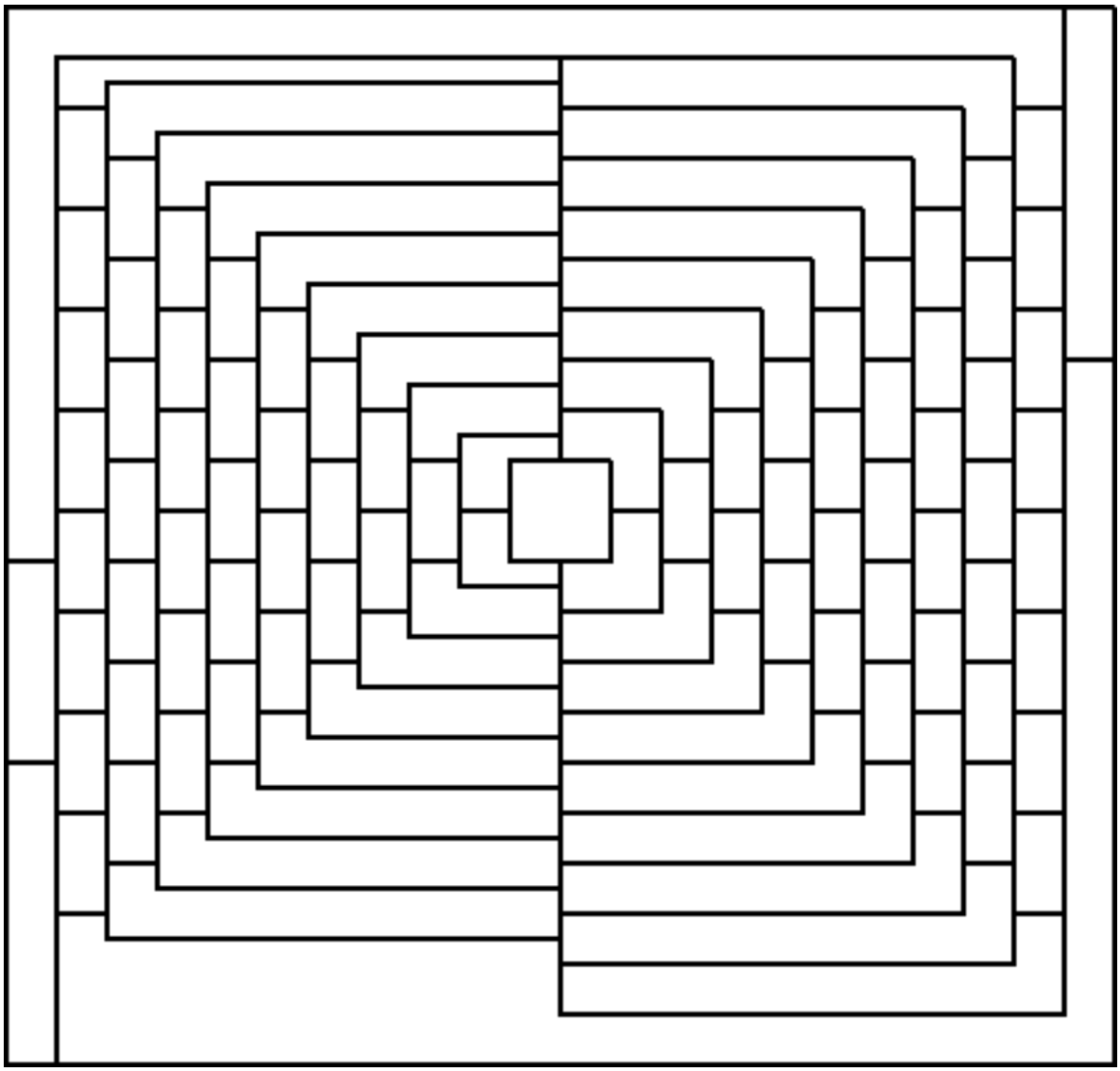
3. Use those two equations to show that, for a planar graph with at least two edges, $E \leq 3V - 6$. Can you use this to show that either of the graphs below aren't planar?



4. Show that a planar graph with at least 3 vertices and no cycle of length three satisfies the inequality $2E \geq 4F$. (A cycle is a path from a vertex back to itself.) Use it to show that, in this case, $E \leq 2V - 4$. Does this help you to decide if any of the above graphs aren't planar?

5. Show that a planar graph with at least three vertices has a vertex with five or fewer edges coming out of it.

6. Rectangularia is divided into 50 counties, as in the picture below. The people of Rectangularia are trying to create a map, and they want to color it with as few colors as possible, but where counties that share a border are always different colors. Help them find a way to color their map. (We have more copies of the map if you need more)



7. Remember from your homework that a *coloring* of a graph is a way of coloring the vertices of the graph so that if two vertices are connected by an edge, they are different colors. Show that you can color any planar graph with six colors.

8. What does coloring a graph have to do with coloring a map?

9. Twenty five boys and twenty five girls are sitting on a round table. Is it possible that every person is sitting next to a boy and a girl?

10. Can you fill in the expression 1__2__3__4 = 0 with + or - signs to make it true? What about the expression 1__2__3__4__5__6__7__8__9__10 = 0?

11. A zoo has 10 red chameleons, 11 yellow chameleons, and 12 blue chameleons. Whenever two chameleons of different colors meet, they turn into the third color. For example, if a red and yellow chameleon meet, they turn blue.

    Is it possible that eventually all the chameleons become the same color?