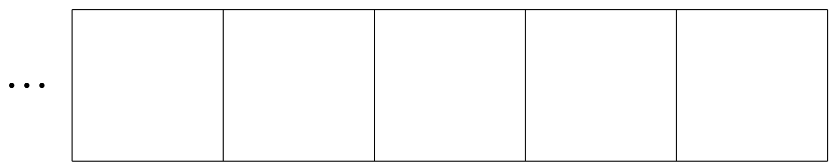
UW Math Circle

November 9th, 2017

1. Remember our exploding dot machines from a few weeks ago. We had a row of boxes, stretching infinitely far in the left direction. When we put some dots in the right most dot, and if we had a $1 \leftarrow n$ machine, any time n dots were together they would explode to left one dot in the next box to the left.



   To begin with, let's consider a $1 \leftarrow 7$ machine.

   (a) If you start with 8 dots in a $1 \leftarrow 7$ machine, how many dots are left in the first square when the explosions are done?

   (b) What about if you instead start with 43 dots?

2. (a) You have a $1 \leftarrow 7$ machine, and you put 32 dots in the first square. How many remain at the end?

   (b) You have a $1 \leftarrow 7$ machine, and you put 82 dots in the first square. How many remain at the end?

   (c) You take the dots remaining in the first square from part a and the dots remaining in the first square from part b, and add them together in a new $1 \leftarrow 7$ exploding dot machine. How many dots remain in the first square at the end?

   (d) Now, you put $32 + 82$ dots in a $1 \leftarrow 7$ machine. How many dots remain in the first square at the end?

   (e) Do you answers to c and d agree? Explain why or why not.

3. A few weeks ago we came up with a method to write a number in base n, where n was an integer. I'll demonstrate our method with an example. Say I want to write 199 in base 5. First, I determine the largest power of 5 that does into 199. This is 5^3, or 125, and 125 goes into 199 once. I then subtract 125 from 199, giving me 74. The biggest power of 5 that goes into this is $25 = 5^2$, and it goes in twice. I subtract $2 * 25$ from 74, giving me 24. The biggest power of 5 that goes into here is $5 = 5^1$, and it goes in 4 times. I'm left with 4, and 5^0 goes into 4 twice.

   So, $199 = 1 * 5^3 + 2 * 5^2 + 4 * 5^1 + 4 * 5^0$, which tells me that in base 5, 199 is 1244. This method makes us determine the leftmost digit of the number first.

   Come up with a new method to determine how to write a number in base n, where you first determine the rightmost digit.

4. Modular arithmetic practice! Fill in the blank with the smallest non-negative integer that satisfies the equation.

   (a) $6 \equiv$ ___ mod 4

   (b) $-14 \equiv$ ___ mod 4

   (c) $43 \equiv$ ___ mod 4

   (d) $80 \equiv$ ___ mod 4

   (e) $163 \equiv$ ___ mod 4

   (f) $80 \cdot 163 \equiv$ ___ mod 4

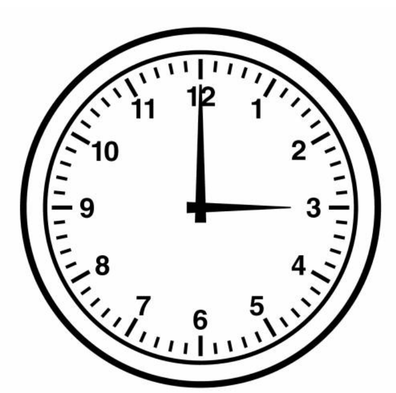
   (g) $9 \equiv$ ___ mod 7

   (h) $22 \equiv$ ___ mod 7

   (i) $75 \equiv$ ___ mod 7

   (j) $2 \cdot 22 + 4 \cdot 75 \equiv$ ___ mod 7

   (k) $(n + 1)^2 \equiv$ ___ mod n



5. Show that $n^3 + 2n$ is always divisible by 3.

6. Show that a number is divisible by 4 if and only if its last two digits are divisible by 4.

7. Show that a number is divisible by 9 if and only if the sum of its digits is divisible by 9.

8. What is the last digit of 2016^{2016}? What about 2017^{2017}?

9. What day of the week will it be 200017 days from today?

10. Figure out a criteria to determine than an odd integer n cannot be written as $a^2 + b^2 = n$, where a and b are integers.