UW Math Circle

November 16, 2017

1. Fill in the following blanks with the smallest non-negative integer satisfying the equation. (Remember, $x \equiv y \mod n$ if x and y have the same remainder when divided by n.)

   (a) $7 \equiv$ ___ mod 3

   (b) $23 \equiv$ ___ mod 5

   (c) $76 \equiv$ ___ mod 14

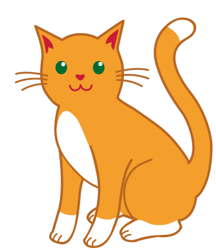
   (d) $-3 \equiv$ ___ mod 5

   (e) $2n + 1 \equiv$ ___ mod 2

   (f) $2n + 1 \equiv$ ___ mod n

2. What are the last two digits of 7^{1942}?

3. Chloe the cat knocked a jar of cat treats off of a shelf. She tried to divide the treats evenly between herself and her cat friends, but when she divided the treats into two piles, there was one left over. When she divided the treats into three piles, four piles, or five piles, there was still one left over each time. If Chloe knows there was no more than 100 treats in the jar, can she determine exactly how many treats there were?



4. Show that $n^5 - n$ is always divisible by 30.

5. For each of the problems below, can you fill in the blank with some number to make the equality true?

   (a) $2 \cdot \_\_\_ \equiv 1 \mod 5$

   (b) $10 \cdot \_\_\_ \equiv 1 \mod 13$

   (c) $5 \cdot \_\_\_ \equiv 1 \mod 7$

   (d) $6 \cdot \_\_\_ \equiv 1 \mod 9$

   (e) $6 \cdot \_\_\_ \equiv 1 \mod 10$

   (f) $6 \cdot \_\_\_ \equiv 1 \mod 11$

   (g) $6 \cdot \_\_\_ \equiv 1 \mod 12$

   (h) In general, can you develop a rule for determining when we can find a solution to $m \cdot \_\_\_ \equiv 1 \mod n$?

6. Notice that $7 \equiv 1 \mod 2$ and $7 \equiv 1 \mod 3$, and also that $7 \equiv 1 \mod 2 \cdot 3$. Also notice that $44 \equiv 2 \mod 3$ and $44 \equiv 2 \mod 7$, and that $44 \equiv 2 \mod 3 \cdot 7$. However, $14 \equiv 2 \mod 6$ and $14 \equiv 2 \mod 4$, but 14 is not equivalent to $2 \mod 4 \cdot 6$.

   When does $a \equiv b \mod c$ and $a \equiv b \mod d$ imply that $a \equiv b \mod c \cdot d$?.