UW Math Circle

November 30, 2017

1. You have four dogs, and you have to choose two of them to take on a walk. How many different combinations of dogs can you take on a walk?

2. You have six bracelets, and you want to choose two of them to put on your arm. How many different ways can you do this? (You care about the order in which you put on the bracelets).

3. (a) You are picking players for your dodgeball team. There are five people you can choose from, and you have to choose three of them. How many ways can do you this?

   (b) What if instead you can choose two of the five?

4. You have four distinct rocks. How many ways can you choose:

   (a) No rocks?

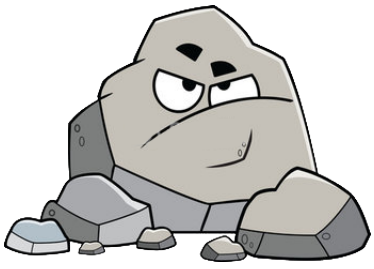
   (b) One rock?

   (c) Two rocks?

   (d) Three rocks?

   (e) Four rocks?

5. Again, you have four distinct rocks. You want to choose some number of rocks from these four. How many different ways can you do this?



6. Compare your answers to the two previous questions.

7. Come up with a formula for the number of ways that you can choose m things out of n things.

8. Pascal's triangle is a big pyramid where the n^{th} row has $n+1$ spots. On the m^{th} spot in the n^{th} row is the number of ways you can choose m things out of n things (we start counting from zero). Here are the first few rows.

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

Draw the next row.

9. Describe as many patterns as you can that you see in the triangle. Try to show why each pattern is true. Try to justify each pattern using our formula for $\binom{n}{k}$ and also by using a counting argument.

10. (a) Show that: $\binom{n}{k} = \binom{n}{n-k}$.
    (b) Show that $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots \binom{n}{n} = 2^n$.