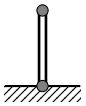
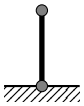
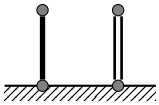
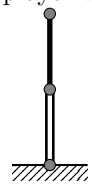
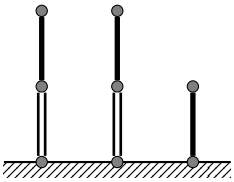
1 Review

Last time we made up a definition for the "tactical value" of a tree. It had four axioms:

1. A tree has a positive value if double edge wins, even if they go first.
2. A tree has a negative value if single edge wins, even if they go first.
3. A tree has a value of zero if the player who goes second always wins.
4. The values of trees should add.

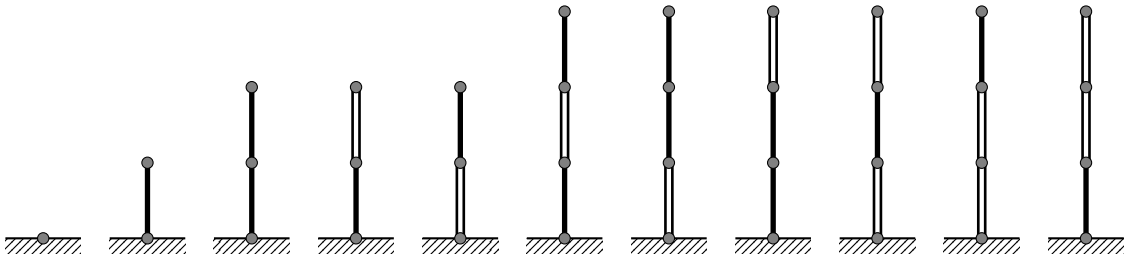
We then decided to give the tree  a value of 1. From this we can prove that  has value -1 because second player wins the game  .

We proved that  has a value of $1/2$ because  is a game that the second player always wins.

2 Problems

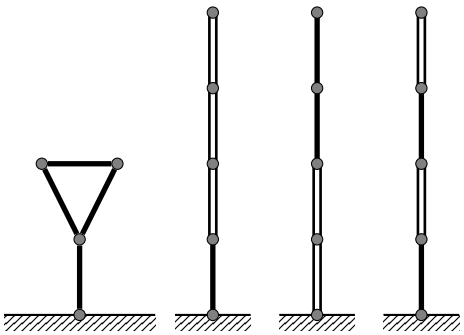
(a) What is the "negative" of a tree?

(b) Here are some **palm trees**.



Suppose you are the black (single-edge) player. If you could add one of them to a game, which would you prefer? Rank them from best to worst.

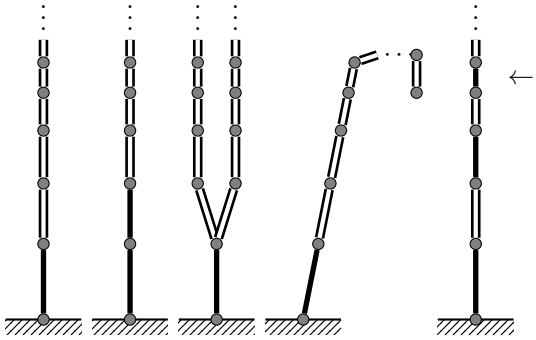
(c) Find the values of the following trees:



Try other examples of taller trees. Is there a quick way to find the value of a skinny, tall tree like these?

(d) How does the value of a tree game change as both players make moves? Try keeping track while playing a few games. Try the same tree with different players going first or second. Can you determine a trees value based on the moves each player can make?

(e) Find the values of these trees:



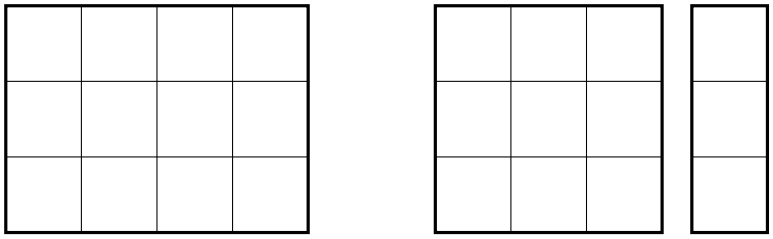
If you don't think these values have a name, give them a name and describe how they compare to other values you do know!

(f) Let's say I give you two complicated trees that we haven't found the value of yet. Even when you don't know exact the value these trees have, can you make a definition for what it means for them to have "equal" value? How about "less than" and "greater than"?

(g) In the tree game we have seen that it is possible for single edge to win, double edge to win, or for second player to win. Is it possible for the first player to win?

(h) Jacob and Lucas are tired of cutting edges on trees and decide to bake a pan of brownies.

But before they can dig in, their competitive spirit takes over again and they decide to make a game out of cutting up the pieces. The players take turns cutting a slice along a gridline all the way across one piece (Jacob horizontally, Lucas vertically), and then separating the resulting pieces. For instance, Lucas might cut the piece on the left into the two pieces on the right:



A player who can't make a cut on their turn loses.

Analyze these games. Can you find "tactical" values for different rectangles?

(i) Is it possible to combine the tree game with the brownie cutting game? Can we combine these games with Nim? How do you add two game in general? What is the value of the sum of games?