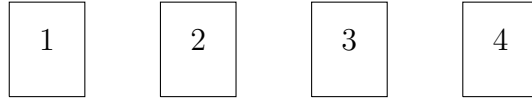


UW Math Circle

1. Place 4 cards in a row, labeled 1 through 4.



- (a) If you are only allowed to switch the position of two cards at a time, can you get all permutations of the numbers 1 through 4? If not, write down what permutations you can get.
- (b) If you are only allowed to switch the position of two *adjacent* cards at a time, can you get all permutations of the numbers 1 through 4? If not, write down what permutations you can get.
- (c) If you are only allowed to switch the position of *two pairs* of two cards at a time, can you get all permutations of the numbers 1 through 4? If not, write down what permutations you can get.
- (d) If you have n cards instead of 4 cards, do your answers to parts (a) through (c) change? Explain why or why not.

2. (a) You have an equilateral triangle in the plane, and you label the vertices 1, 2, 3. You can move the triangle by rotating it and by flipping it over, but after you are finished it must fit back exactly where it started, but there may be a different ordering on the vertices. How many permutations of one, two, and three can you get by doing this? Write your answers in cycle notation. Do you get all of them?
- (b) You do the same thing, except now you start with a square. What permutations of one, two, three, four can you get? Do you get all of them?
- (c) Instead of allowing any movement of the square, let's say the only allowed operations on the square are the 90 degree clockwise rotation (P) and the reflection around the vertical axis (Q). If we can do these operations as many times as we want, and in whatever order we want (so, for example, we could do $QPPPPQP$), what permutations can we get? Do we get the same answer as in part (b)?
- (d) What about with a regular n -gon? Using the cycle notation for permutations, find a compact way to write the possible permutations you can get. (You may find it helpful to try it for a few other shapes, like a regular pentagon or a hexagon. If this seems hard, can you instead figure out how many permutations you get?)