UW Math Circle

- 1. Radagast the Brown's Ice Cream Shack offers 5 different flavors of ice cream.
 - (a) In how many ways can you make a sundae with 3 scoops of ice cream if you want three different flavors?
 - (b) In how many ways can you make a sundae with 3 scoops of ice cream if you're allowed to order multiple scoops of the same flavor?
- 2. Because of his skills as an ice cream maker, Radagast decides to expand his line of frozen treats to offer 7 different flavors of ice cream. In how many ways can you make a sundae with three different scoops of ice cream now?



- 3. Is there a connection between the number of 3-scoop sundaes with 5 possible flavors (with possible repeats of flavors) and the number of 3-scoop sundaes with 7 possible flavors (with no repeats of flavors)?
- 4. What is the sum

$$\binom{n}{0} + \binom{n-1}{1} + \binom{n-2}{2} + \binom{n-3}{3} + \cdots,$$

where you will stop adding when the binomial coefficients don't make sense any more. For example, when n = 9, this will be the sum:

$$\binom{9}{0} + \binom{8}{1} + \binom{7}{2} + \binom{6}{3} + \binom{5}{4},$$

but the next term, $\binom{4}{5}$ doesn't make sense because we can't choose 5 numbers from 4 possibilities.