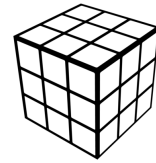


UW Math Circle  
Week 3

1. How many diagonals can be drawn in a triangle? In a square? In an octagon? In an  $n$ -gon?

2. A thousand unit cubes are fastened into one large cube. This large cube is then painted and dissembled. How many unit cubes are painted on at least one side? Can you find two ways to solve this problem?



3. Tom, Bert and William Huggins each roll a die six times. What is the probability that all of the brothers roll at least a five at least five times? What if they each rolled a die seven time? What if they each rolled a die  $n$  times?

4. How many integers strictly between 100 and 900 contain at least one 2? Can you solve this problem without counting how many numbers have a 2?

5. The permutations of the letters M,A,T,H,S are arranged in dictionary order. What is the 53rd word? Try to solve the problem without writing out every word!

6. For any real number  $a$  and positive integer  $n$  define  $a \circ n$  to be

$$a \circ n = \frac{a(a-1)(a-2)\dots(a-(n-1))}{n(n-1)(n-2)\dots(2)(1)}$$

What is  $(\frac{-1}{2} \circ 100) \div (\frac{1}{2} \circ 100)$ ?

7. How many ways can you make a list of  $n$  positive numbers which adds up to  $2n-2$ ?
  
8. Consider an  $n$ -by- $n$  grid. How many ways can you walk from the bottom left to the top right if you are not allowed to cross the diagonal line? What if you have to stay strictly below the diagonal line?