

**Problem 1.** Design a corn maze with seven scarecrows, where every pair of scarecrows is connected by a path, with as few possible crossings of the paths. Shown below is a design with five scarecrows and three crossings.

**Problem 2.** Take a positive number and repeatedly take the sums of the squares of its digits. If this terminates at 1, call this number a decaying number. (For example, 31 is a decaying number because  $31 \rightarrow 10 \rightarrow 1$ ). Find a decaying number less than 10 000 that takes as long as possible to terminate.

**Problem 3.** Find the longest word you can where the letters alternate going forwards and backwards in the alphabet. For example, “FANG” is such a word, because “F” is after “A” in the alphabet, “A” is before “N”, and “N” is after “G”.

**Problem 4.** Frodo is playing a game with Gandalf. Frodo must place red, blue, and yellow pieces on a square grid so that the following is satisfied:

- No two rows or columns are the same.
- If any “L” on the board has the same color at all three corners, then the fourth corner of the rectangle formed by the “L” is also that same color.

Frodo and Gandalf are competing to make the biggest grid possible. Gandalf has made a large grid, but kept it hidden. Frodo shows Gandalf the  $3 \times 3$  arrangement below, but Gandalf says it’s not the biggest and he can beat it. What is the largest board that you can make to try and beat Gandalf?

$$\begin{array}{ccc} R & Y & B \\ B & R & Y \\ B & Y & R \end{array}$$

**Problem 5.** Find a sequence of equally spaced prime numbers that is as long as possible. For example, (5, 11, 17) would be a sequence of length 3, and it is equally spaced because  $11 - 5$  and  $17 - 11$  are both 6.