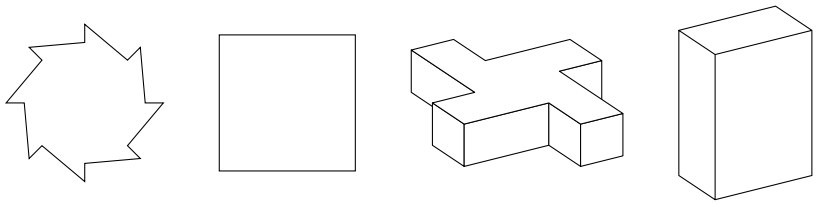
Problem 1. Jabba the Hutt collects mattresses. Here are some of his favourites! The first two are 2D, and the last two are 3D:



Jabba has a Reflexor Engine that can send a 3D mattress to a warp dimension and return a mirror-image version.

What many mattress-flipping actions are there for each mattress, if you're allowed to use the Reflexor as part of the action? How many actions are there for each?

Problem 2. Write down permutation notation for the actions.

Problem 3. These four mattresses all have the same number of flips, but the symmetries are still different. Let's try to find some attributes that distinguish them!

(a) For each mattress, what's the smallest list of actions that you can use, to get every possible action by doing actions on your list in some order?

(b) Some mattress actions can be done in any order. For example, if you rotate by 90° then rotate by 180°, the result is the same as rotating by 180° first then by 90°.

Does Jabba have a mattress where all the actions can be done in any order?

(c) The "order" of a mattress action is the number of times you have to do it before you get back to where you started. For example, the order of rotating by 90° is 4, since doing this action four times has the same result as doing nothing. Which mattresses have an action of order 4? Order 8? Order 2? Order 3?

(d) For every mattress action, there is an action that un-does it and brings you back to what you started with — this is called its "inverse". Sometimes this is a different action than the first one, but sometimes it's the same.

Which actions are their own inverses? Does this have something to do with the previous question?

Does the order matter when you do an action and its inverse?