Last week, we talked about the following four mattresses:


Remember that each mattress has 8 possible mattress flipping actions (including the "do nothing" action, and mirror-imaging for some mattresses).
Problem 1. Write down a "multiplication table" for the mattress actions.
I've already started the table for the first mattress: " $N$ " means the "do Nothing" action, and " 5 " means "rotate clockwise by 5 eighths of a full rotation", for example. The entry in the " $r 3$ " row and " $r 6$ " column is " $r 1$ ", which means "if I rotate by 3 eighths then 6 eighths, it's the same as if I just rotated by 1 eighth".

You'll need to come up with your own notation for the other mattresses!

|  | $N$ | $r 1$ | $r 2$ | $r 3$ | $r 4$ | $r 5$ | $r 6$ | $r 7$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $N$ | $N$ | $r 1$ | $r 2$ | $r 3$ | $r 4$ | $r 5$ | $r 6$ | $r 7$ |
| $r 1$ | $r 1$ | $r 2$ | $r 3$ | $r 4$ | $r 5$ | $r 6$ | $r 7$ | $N$ |
| $r 2$ | $r 2$ | $r 3$ | $r 4$ | $r 5$ | $r 6$ | $r 7$ | $N$ | $r 1$ |
| $r 3$ | $r 3$ | $r 4$ | $r 5$ | $r 6$ | $r 7$ | $N$ | $r 1$ | $r 2$ |
| $r 4$ | $r 4$ | $r 5$ | $r 6$ |  |  |  |  |  |
| $r 5$ |  |  |  |  |  |  |  |  |
| $r 6$ |  |  |  |  |  |  |  |  |
| $r 7$ |  |  |  |  |  |  |  |  |



Problem 2. Do you notice any cool facts about these tables?

Problem 3. Last week, we asked you to figure out some properties of the sets of mattress symmetries. These are the things we asked:

- For each mattress, what's the smallest list of actions you need so that every possible action can be done by doing things on your list in some order?
- Are there any mattresses where it doesn't matter what order you do the actions in?
- How many times do you have to do each action before you get back to where you started?
- For each action, there's something that un-does it. This is sometimes the same action and sometimes a different one - when is it the same?

Do the tables help answer any of these questions?

