## UW Math Circle <br> Worksheet

1. Kristoff and Sven have invented a new game to pass the time during the summer. They stack $n$ coins into a single pile and take turns taking any number of coins from the pile (but they have to take at least one!). The loser is the player who takes the last coin. If Sven goes first, what strategy should he use?
2. Kristoff soon realized that whenever Sven went first, Sven won. Convinced he was cheating, Kristoff added new rules to the game. Now there are three piles, each with any number of coins. The game is played the same as before but each turn the player can only take coins from one stack. Kristoff lets Sven go first. What strategy should each player use? Can Kristoff or Sven find a strategy to always win? Try playing the game with a partner!
3. If Kristoff adds a fourth pile, do their strategies change? What about a fifth pile? Or a sixth?
4. Sven decides to add his own rules to the game. Coins are stacked into any number of piles, each of any size. Players can take any number of coins from any number of piles, as long as they take the same number from each pile (and they take at least one coin). If Sven still goes first, should he change his strategy? What about Kristoff?
5. Wanting to brush up on his math skills, Sven adds the rule that each player can only take a prime number of coins from each pile. The loser is the player that takes the last coin or leaves a board with no viable moves. Does this change what strategy each player should use?
