UW Math Circle

Jigglypuff has come up with a new way of writing down numbers. Instead of using numbers, he is going to use capital letters. He started to write out a guide for translating from our numbers to his, but soon stopped.

$$
\begin{aligned}
\text{A} &= 0 \\
\text{B} &= 1 \\
\text{C} &= 2 \\
\text{D} &= 3 \\
\text{E} &= 4 \\
\text{F} &= 5 \\
\text{G} &= 6 \\
\text{H} &= 7 \\
\text{I} &= 8 \\
\text{J} &= 9 \\
\text{K} &= 10 \\
\text{L} &= 11 \\
\text{M} &= 12 \\
\text{N} &= 13 \\
\text{O} &= 14 \\
\text{P} &= 15 \\
\text{Q} &= 16 \\
\text{R} &= 17 \\
\text{S} &= 18 \\
\text{T} &= 19 \\
\text{U} &= 20 \\
\text{V} &= 21 \\
\text{W} &= 22 \\
\text{X} &= 23 \\
\text{Y} &= 24 \\
\text{Z} &= 25 \\
\text{AA} &= 0 \\
\text{AB} &= 1 \\
\text{AC} &= 2 \\
\text{BA} &= 26 \\
\text{BB} &= 27 \\
\text{BC} &= 28 \\
\text{CA} &= 52 \\
\text{CB} &= 53 \\
\text{DA} &= 78 \\
\text{EA} &= 104 \\
\text{FA} &= 130 \\
\text{AZ} &= 25
\end{aligned}
$$

$$ZA = 650$$
$$AAAAA = 0$$
$$A + B = B + A = C$$
$$B + Z = BA$$
$$A*B = B*A = 0$$
$$B*ONE = ONE$$
$$ZERO = 442560$$
$$ONE = 9806$$
$$TWO = 13430$$

Can you figure out what ONE + TWO should be in Jigglypuff numbers? What about JIGGLYPUFF? Or your name? Can you write a mathematically and/or grammatically correct sentence using Jigglypuff numbers?

1. Pick three random distinct integers between 1 and 9. Can you write every number from 0 to 10 as a combination of those integers using only addition, subtraction, multiplication, division and exponents? What if one of the numbers is 0? What if two of the numbers are the same? What if all three are the same number?

2. Can you find a number x such that x is a square, $x + 1$ is a cube and $x + 2$ is a quartic?

3. At Poke-center, there is usually a track for the 60-meter dash. It also has a starting line for a 50-meter dash with respect to the same finish line. In training, Vulpix wanted to run 30 meters, but there was no line for that. What is the minimum number of additional lines that need to be added so that every distance meters can be run on this track?

4. The sum of the ages of my five Eevee evolutions is 47. Their ages are positive integers, and any two of them have a common divisor greater than 1.

   How old is the eldest?

5. Recall that a magic square consists of a grid filled in with the digits 1-9 such that every column, row, or diagonal adds up to the same constant (typically 15). Is it possible to fill each box with a distinct integer such that the product of every column, row, or diagonal is the same number?

6. Can you find a positive integer $n > 1$ such that $n!$ is a perfect square?