

UW Math Circle

In his spare time, Mew collects necklaces. However, Mew only collects very specific types of necklaces. To Mew, a necklace is a distinct strand of p different colored beads in up to n colors, where p is a prime number. Mew also requires that every necklace is unique under rotation. For example, the two necklaces below are the same:

but the two necklaces below are different:

Jealous of Mew, Mewtwo decides to cut all of Mew's necklaces into strands. Mewtwo offers to fix all of the necklaces if Mew can determine a prime divisor of the number of necklaces cut without counting the strands. Mew, a mathematical prodigy, starts to jot down answers to the following questions.

1. How many unique strands of p beads in up to n colors exist?
2. How many strands of length p with at least two colors exist?
3. How many necklace strands of length p in at least two colors come from the same necklace?

After answering the questions above, Mew correctly answers Mewtwo's question. What was Mew's answer?