## UW Math Circle

Homework

1. The country of Uzbekikazakhturkistan has 9 major cities, which are called $1,2,3$, $4,5,6,7,8,9$. The UHS (Uzbekikazakhturkistani Highway System) connects city A and city B with a road provided that the two digit number AB is divisible by 3 . Is it possible for Brave Sir Cosmo to travel by road from any one city to any other city? [For example, he can get from city 1 to city 7 by traveling $1 \rightarrow 5 \rightarrow 7$, even though there isn't a direct road from city 1 to city 7.]
2. In a large city park, there are 239 benches, and the distances between all pairs of benches are different. Secret agents $001,002, \ldots, 00239$ are all sitting on a different bench, and each agent is spying on the agent who is sitting on the closet bench to his/her own. Show that there is some agent that no one is spying on.
