UW Math Circle Homework

1. The country of Uzbekikazakhturkistan has 9 major cities, which are called 1, 2, 3, 4, 5, 6, 7, 8, 9. The UHS (Uzbekikazakhturkistani Highway System) connects city A and city B with a road provided that the two digit number AB is divisible by 3. Is it possible for Brave Sir Cosmo to travel by road from any one city to any other city? [For example, he can get from city 1 to city 7 by traveling $1 \rightarrow 5 \rightarrow 7$, even though there isn't a direct road from city 1 to city 7.]

2. In a large city park, there are 239 benches, and the distances between all pairs of benches are different. Secret agents 001,002,...,00239 are all sitting on a different bench, and each agent is spying on the agent who is sitting on the closet bench to his/her own. Show that there is some agent that no one is spying on.