Question 1. Are the following numbers even or odd?

(a) 1234×567

(b) $45836 \times 7823 + 89273 \times 9231$

(c) 2^{84}

(d) 17^{429}

(e) $10^{10^{10}}$

(f) $23^{23^{23}}$

(g) $1^1 + 2^2 + 3^3 + \cdots + 9^9 + 10^{10}$

(h) $129!$

(i) The 100th Fibonacci number

(j) The 999th triangle number

Question 2. What is the last digit of each of the numbers from question 1?

Question 3. Which of these numbers are divisible by 3?

The three questions on the previous page can be answered in a similar way.

- The last digit of a number is the same as the remainder when you divide the number by 10.
- The remainder when you divide by 2 is 1 if the number is odd and 0 if it's even.
- A number is divisible by 3 whenever the remainder when you divide by 3 is 0.

If two numbers a and b have the same remainder when you divide them by q, then we say that "a and b are congruent modulo q", written "$a \equiv b \mod q$". For example, 36 is congruent to 86 modulo 10, since 36 and 86 both have the same remainder (namely 6) when you divide them by 10. Here are some more examples:

$$12 \equiv 5 \mod 7, \qquad 124 \equiv 54 \mod 10,$$
$$38 \equiv 0 \mod 2, \qquad 100 \equiv 898 \mod 3.$$

Question 4. True or false:

(a) $843643538 \equiv 5345636 \mod 10$

(b) $843643538 \equiv 5345636 \mod 2$

(c) $843643538 \equiv 5345636 \mod 5$

(d) $843643538 \equiv 5345636 \mod 3$

(e) $843643538 \equiv 5345636 \mod 9$

(f) $843643538 \equiv 5345636 \mod 57$

Question 5. Suppose that when you divide a by q, the remainder is r, and when you divide b by q the remainder is s. Are the following equations true? Try to explain why they're true, if you can, or find some numbers for a, b and q that break things.

(a) $a + b \equiv r + s \mod q$. (For example, $10004 + 10003 \equiv 4 + 3 \mod 10$.)

(b) $a \times b \equiv r \times s \mod q$

(c) $a^b \equiv r^s \mod q$

On the previous worksheet, you should have figured out that

$$a + b \equiv r + s \mod q$$

where r and s are the remainders of a and b when you divide by q. This means that if you want to find the remainder of $a + b$ when you divide by q, you don't actually need to add a and b: you can take the remainders of a and b first, then just add the remainders together. This is useful if a and b are very big numbers, for example.

Question 6. What does this have to do with questions 1 to 3?

Question 7. What is the remainder when you:

(a) ... divide $3256 + 1982$ by 10?

(b) ... divide $(703 + 356 + 79 + 3)$ by 7?

(c) ... divide $(7846217 + 439878142 + 87437632)$ by 100?

(d) ... divide $(3614 + 3614 + 3614 + 3614 + 3614)$ by 36?

(e) ... divide (2802×5) by 14?

(f) ... divide (6793×6) by 8?

(g) ... divide (197×197) by 196?