## University of Washington Math Hour Open Olympiad, 2012 Grades 8-10

**6.** In a certain herd of 33 cows, each cow weighs a whole number of pounds. Farmer Dan notices that if he removes any one of the cows from the herd, it is possible to split the remaining 32 cows into two groups of equal total weight, 16 cows in each group. Show that all 33 cows must have the same weight.



7. Katniss is thinking of a positive integer less than 100: call it x. Peeta is allowed to pick any two positive integers N and M, both less than 100, and Katniss will give him the greatest common divisor of x + M and N. Peeta can do this up to seven times, after which he must name Katniss' number x, or he will die. Can Peeta ensure his survival?