Your Name:______________________________________ **Room:**__________________________

Grade:___________________________________ **Teacher:**________________________________

Montlake Math Challenge

Montlake Elementary School

December 13, 2007

Instructions: Try to answer as many of these problems as you can. Work on your own. If you don't know how to completely solve a problem, try to write as much as you can. You don't have to solve the problems in order, and you are not expected to solve every problem.

Problem 1: Sally has a horse. Three people say things about the color of the horse:

Alan says, "The horse is not black."
Bob says, "The horse is either brown or gray."
Carl says, "The horse is brown."

At least one of the above statements is true, and at least one of the above statements is false. If Sally's horse is either black, brown, or gray, can you decide what the horse's true color is?

Solution: The horse is gray. If the horse were black, then each of the people would be a liar. If the horse were brown, then each of the people would be telling the truth.

Problem 2: You have six coins that all have the same size and shape. Five of the coins have the exact same weight, but the sixth coin is slightly heavier than the other five. You cannot tell which coin is heavier by just looking at them or holding the coins in your hands. You also have a balance scale. Show that it is possible to find out which coin is heaviest by only using the balance scale **two** times.

Solution 1: Break the coins into two piles of three coins each. Put each pile on one side of the balance, and decide which side contains the heavier coin. Now pick any two of the coins from the heavier side and weigh them. If one of the coins is heavier than the other, it is the heaviest coin. If both of the coins weigh the same amount, then the third coin that wasn't weighed is the heaviest one.

Solution 2: Break the coins into three piles of two coins each. Pick two piles and put one on either side of the balance. If one of the piles is heavier, put one of its coin on either side of the balance to determine which of the coins is the heaviest. If both piles have the same weight, then the heaviest coin must be in the pile that you didn't weigh. Weigh those two coins against one another to decide which is the heavier one.

Problem 3: If I have any three numbers, show that I can pick two of them whose sum is even. (For example, if my numbers are 3, 8, and 7, then 3+7 = 10, which is even.)

Solution: If I have three numbers then there must be either (a) at least two even numbers or (b) at least two odd numbers. The sum of two even numbers is even, and the sum of two odd numbers is even.

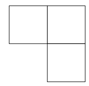
Problem 4: Some bacteria are placed in a glass. Every second the number of bacteria in the glass doubles. After one minute, the glass is full of bacteria. How long did it take for the glass to be half full?

Solution: This problem is actually very tricky. It only takes one second for the glass to go from halfway full to completely full. Since it takes one minute for the glass to be completely full, it takes **59** seconds for the glass to be half full.

Problem 5: I have four numbers. Show that there are two of the numbers whose difference is divisible by three. (Hint: If I divide a number by three, how many possible remainders are there? First show that two of my four numbers have the same remainder when I divide them by three.)

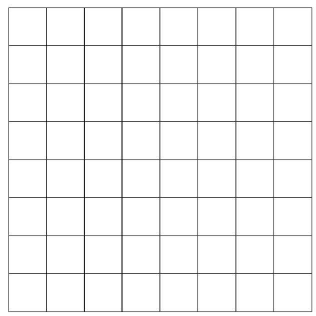
Solution: If I divide a number by three, there are three possible remainders: 0, 1, or 2. Since I have four numbers, two of them must have the same remainder when I divide them by three. The difference of these two numbers is divisible by three.

Problem 6: You have a bunch of L-shaped dominoes that look like this:



Is it possible to cover each of the following checkerboards with L-shaped dominoes that do not overlap? Why or why not?

a)



This board cannot be covered by the L-shaped dominoes pictured above. The board has 64 squares, and each domino covers 3 squares. Since 64 is not divisible by 3, the board cannot be covered.

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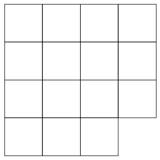
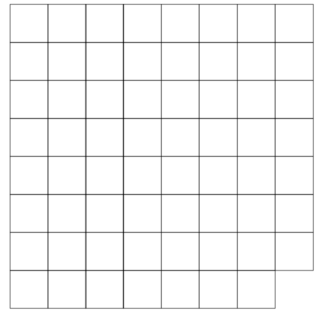
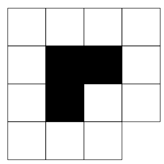
b)



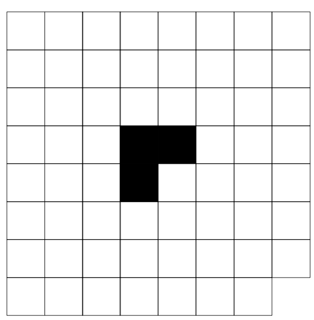
c)



Board (b) can be covered by the L-shaped dominoes. The shaded region shows where to put one domino, and then we place four more dominoes around the perimeter.



Board (c) can also be covered by the L-shaped dominoes. The shaded region shows where to put the first domino. The the rest of the board looks like four copies of board (b), which we already know can be covered by L-shaped dominoes.



Problem 7: A slug is trapped at the bottom of a well that is 30 inches deep. Every day, he climbs 3 inches up the side of the well, and every night he slides 2 inches back down the well. How long does it take him to get out of the well? Why?

Solution: This problem is also tricky. On the morning of the second day, he will have climbed one inch up the side of the well. On the morning of the third day, he will have climbed two inches up the side of the well. On the morning of the fourth day, he will have climbed three inches up the side of the well, etc, etc. On the morning of the 28^{th} day, he will have climbed 27 inches up the side of the well. On the morning of the 29^{th} day, he will have climbed 28 inches up the side of the well. But now, during the 29^{th} day, he will climb three inches up the side of the well, and he will escape from the well.

The answer is **29** days.

Problem 8: Alice and Dana play a game in which they take turns placing quarters on a table that has the shape of a circle. The only rule for the game is that the quarters cannot overlap. The last person who can place a quarter wins all the money on the table. If Alice goes first, show that she can always win the game.

Solution: This problem is also quite tricky. Alice can win by placing her first quarter at the center of the table. On every turn she makes after that, she places a quarter in the exact opposite position as Dana places her quarter. For example, if we think about the table as a clock and Dana places a quarter 8 inches from the center of the circle at 8:00, then Alice should respond by placing her quarter 8 inches from the center of the circle at 2:00.

Alice wins because she can always respond to any legal move that Dana can make.