$\qquad$
$\qquad$
Grade: $\qquad$ Teacher: $\qquad$

# Montlake Math Challenge <br> Montlake Elementary School 

February 7, 2008

Problem 1: Fill in the $(\bmod 5)$ addition table below:

|  | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |

Problem 2: Fill in the $(\bmod 8)$ addition table below:

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  |  |

Problem 3: Fill in the blanks.
a) $8(\bmod 3)=$ $\qquad$
b) $17(\bmod 8)=$ $\qquad$
c) $8(\bmod 4)=$ $\qquad$
d) $7(\bmod 5)=$ $\qquad$
e) $6(\bmod 5)=$ $\qquad$
f) $7+6(\bmod 5)=$ $\qquad$
g) $2+1(\bmod 5)=$ $\qquad$
h) $20(\bmod 11)=$ $\qquad$
i) $35(\bmod 11)=$ $\qquad$
j) $20+35(\bmod 11)=$ $\qquad$
k) $9+2(\bmod 11)=$ $\qquad$

1) $7(\bmod 3)=$ $\qquad$
m) $5(\bmod 3)=$ $\qquad$
n) $7 \times 5(\bmod 3)=$ $\qquad$
o) $5(\bmod 4)=$ $\qquad$
p) $5 \times 5(\bmod 4)=$ $\qquad$
q) $5 \times 5 \times 5(\bmod 4)=$ $\qquad$
r) $10(\bmod 6)=$ $\qquad$
s) $9(\bmod 6)=$ $\qquad$
t) $10 \times 9(\bmod 6)=$ $\qquad$
u) $4(\bmod 5)=$ $\qquad$
v) $4 \times 4(\bmod 5)=$ $\qquad$
w) $4 \times 4 \times 4(\bmod 5)=$ $\qquad$
x) $4 \times 4 \times 4 \times 4(\bmod 5)=$ $\qquad$

Problem 3: Fill in the blanks.

1. $\operatorname{gcd}(4,6)=$ $\qquad$
2. $\operatorname{gcd}(21,12)=$ $\qquad$
3. $\operatorname{gcd}(50,60)=$ $\qquad$
4. $\operatorname{gcd}(9,27)=$ $\qquad$
5. $\operatorname{gcd}(7,9)=$ $\qquad$
6. $\operatorname{gcd}(9,33)=$ $\qquad$

Problem 4: A prime number is a number $p$ whose only divisors are 1 and $p$. For technical reasons, we do not allow 1 to be a prime number.

Problem 4a: Write the first eight prime numbers.

Problem 4 b : If $p$ is a prime number and $n$ is any other number, what are the possible values for $\operatorname{gcd}(p, n)$ ?

Problem 5: In the following table, fill in the gcd of the listed number with 12 :

| Number $n$ | $\operatorname{gcd}(n, 12)$ |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |
| 11 |  |
| 12 |  |

Problem 6: Given a number $n$, we say its order $(\bmod 12)$ is the smallest number of times we can add $n$ to itself to get a number that is divisible by 12 . For example, if $n=8$, we have $8+8=16$, which is not divisible by 12 $8+8+8=25$, which is divisible by 12
so the order of 8 is 3 . Fill in the table below with the orders of each listed number (mod 12 ).

| Number $n$ | Order of $n(\bmod 12)$ |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |
| 11 |  |
| 12 |  |

Problem 7: Fill in the following table using problems 5 and 6 . What do you notice?

| Number $n$ | $\operatorname{gcd}(n, 12)$ | Order of $n(\bmod 12)$ |
| :--- | :--- | :--- |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |

