Divisibility I

Odd and Even or Divisibility by Two.

Problem 1. Can a 5×5 square checkerboard be covered by 1×2 dominoes? How or why not?

Problem 2. Can one make change for a fifty dollar bill with ones and fives using in all fifteen bills? How or why not? How about using a total of twenty-one bills?

Problem 3. Pete bought a notebook containing 96 pages numbered 1 through 192. Harry tore out 25 pages from Pete's notebook and added the 50 numbers that he found on those pages. Could Harry have gotten 2010 as the sum?

Problem 4. The numbers 1 through 10 are written in a row. Can you insert the signs '"+'" and "−" between them so that you produce an expression whose value is zero?

Divisibility by 3, 4, 5, 7, 9, and 11.

Problem 5. How can you quickly determine whether a number is divisible by 2 or by 5? More importantly, why does this work?

Problem 6. How can you reasonably quickly determine whether a number is divisible by 4? Can we do something a bit like what we do for 2 and 5?

Problem 7. How can you quickly determine whether a number is divisible by 3? And, this is harder, why does this work? Is there a similar approach to divisibility by 9?

Problem 8. How can you quickly determine whether a number is divisible by 11?

Problem 9. Now for the hard one, how can you reasonably quickly determine whether a number is divisible by 7? (The approach is related to the approach for 11.)