## Geometry Puzzles

Problem 1 (But first, one last logic puzzle). Suppose that you are a prisoner, and you are confronted with two doors: one leading to freedom, and one leading to the executioner's chamber, but you don't know which is which. A sentry guards each door. You know that one sentry always lies, and one sentry always tells the truth. Again, you don't know which is which. You are allowed to ask one of the sentries one question. What is a question you ask a sentry that will tell you with certainty which door leads to freedom?

Problem 2. Toothpicks are arranged as shown below. Move two toothpicks to form four squares each with side length equal to one toothpick.


Figure 1: Toothpicks, move two.

Problem 3. Is it possible to arrange six pencils so that each pencil touches each of the others? If so, how?

Problem 4. Toothpicks are arranged as shown below. Remove four toothpicks to leave two equilateral triangles. Remove three toothpicks to leave, again, two equilateral triangles. Finally, remove just two toothpicks to leave two equilateral triangles. Loose ends are not allowed.


Figure 2: Toothpicks, remove four, three or two, leaving two equilateral triangles.

Problem 5. Ten pennies are arranged as shown below. What is the minimum number of pennies we must remove so that no three of the remaining pennies lie on the vertices of an equilateral triangle.


Figure 3: Pennies.

Problem 6. Draw six lines segments to form eight triangles. See if you can find two different ways of accomplishing this.

Problem 7. A river that is 4.0 meters wide makes a $90^{\circ}$ turn as shown below. Is it possible to cross the river by bridging it with only two planks laid flat, each 3.9 meters long. If so, how?


Figure 4: River and Planks.

Problem 8. In the distant future, regular travel between planets will become possible. Suppose spacecraft travel along the following routes: Earth-Mercury, Pluto-Venus, Earth-Pluto, PlutoMercury, Mercury-Venus, Uranus-Neptune, Neptune-Saturn, Saturn-Jupiter, Jupiter-Mars, and Mars-Uranus. Can a traveler get from Earth to Mars?

Problem 9 (Think about using graphs). A chessboard has the form of a cross, created from a $4 \times 4$ chessboard by deleting the corner squares. Can a knight travel around this chessboard, using the usual knight's move, passing through each square exactly once, and end up on the same square it starts on?


Figure 5: Knight's tour.

Problem 10 (Again, think about using graphs). Four knights are positioned on a $3 \times 3$ chessboard as shown on the first chessboard below. Can they move to the positions shown on the second chessboard?


Figure 6: Knight's positions, before and after.

