Parity or Paired Things

Left-Right, Black-White, In-Out

Problem 1. Seven gears are arranged on a plane as shown below. Can all the gears rotate simultaneously?
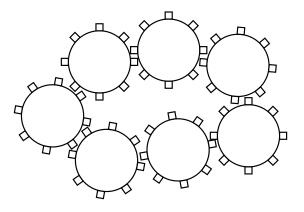
Figure 1:

Problem 2. Remembering that a knight's move in chess is straight two squares and over one, show that if a knight starts in one corner of a chessboard and after several moves returns to it, then it must have made an even number of moves.

Problem 3. Now suppose a knight starts in one corner of an 8×8 chessboard. Can it through a series of moves visit each square exactly once and end up in the corner diagonally opposite the one it started in? Why or why not?

Problem 4. A closed path is made up of seven line segments. Can one line that doesn't touch a vertex of the path intersect each of its segments? How or why not? Hint: try answering the question for a three segment and a four segment closed path first.

Problem 5. Can we draw a closed path made up of nine line segments each of which intersects exactly one other segment.

Odd and Even

Problem 6. Can a 5×5 square checkerboard be covered by 1×2 dominoes? How or why not?

Problem 7. Can one make change for a fifty dollar bill with ones and fives using in all fifteen bills? How or why not? How about using a total of twenty-one bills?

Problem 8. Pete bought a notebook containing 96 pages numbered 1 through 192. Harry tore out 25 pages from Pete's notebook and added the 50 numbers that he found on those pages. Could Harry have gotten 2010 as the sum?