# Problem Set XI Homework

**1.** Prince Ivan is on a quest to free Princess Masha, who has been imprisoned in the castle. The castle door has a simple digital lock with ten buttons that are numbered 0 to 9. Also, the door is guarded by a hungry dragon Pashka who likes hot dogs. The door lock can be opened by typing a secret 4-digit code, and Pashka can be distracted by hot dogs. It takes 1 second for Prince Ivan to try out a single 4-digit combination, and it takes 20 seconds for Pashka to gulp down a single hot dog. After Ivan opens the lock, it will take him one minute to fetch Masha and fly off on his magic carpet.

**(a)** How many hot dogs should Ivan pack for the quest if he wants to fly out of the castle alive and with Masha? (Ivan should have enough hot dogs to keep Pashka at bay even in the worst case: when the secret code will be the last possible combination that he will try.)

**(b)** Suppose that Ivan knows in advance that the secret 4-digit code is composed of odd digits only. How many hot dogs would he need now?

**(c)** Suppose that Ivan knows in advance that the secret 4-digit code is composed of odd digits only and has exactly one digit 5 in it. How many hot dogs would he need in this case?

# Problem Set XI Challenge Problems

**2.** Twenty numbers are placed around the circle in such a way that any number is the arithmetic mean of its two neighbors. Prove that all these numbers are equal.

**3.** A robot drew the following painting: a 4 × 4 white square with the top-left corner cell painted black. You can command the robot to select a row or a column of your choice and repaint every single square in this row (column) into the opposite color.

Can you create a sequence of commands that would turn the entire 4 × 4 square black? Either demonstrate such a sequence or prove that it does not exist.