Math Challenge

Washington Middle School
December 3, 2009

Warm-Up Activity

Congratulations, you have been accepted to Hogwarts! ....almost

Before you can be enrolled in the school of magic and wizardry, you must pass Professor Snape's final test. There are two doors in his classroom. Behind each door, you will find either Harry Potter or Voldemort. Be careful! Harry Potter might be behind both doors or Voldemort might be behind both doors. This is a school of magic and wizardry after all!

If you find Harry Potter, he will teach you how to become the best wizard in the world! If you find Voldemort...you probably don't want to find Voldemort.

Professor Snape will put signs on each door, and you will have to use logical reasoning to try to find Harry Potter!

Problem 1: Professor Snape tells you that either both signs are true or both signs are false. Which door should you open?

Door 1	Door 2
Either Voldemort is in this room or Harry Potter is in the other room.	Harry Potter is in the other room.

Problem 2: Seven people who voted in last year's presidential election are in a room. Show that there are at least four people who voted for the same candidate. (Assume that Obama and McCain were the only candidates)

Problem 1: There are 370 students at Mathland Elementary School. Show that there are two of them who have the same birthday.

Problem 2: Choose five numbers: u,v,w,x,y. In this problem, we will show that there are two numbers among $2^u, 2^v, 2^w, 2^x, 2^y$ whose difference is divisible by 10.

For example, if

$u=1$	$2^u=2$
$v=2$	$2^v=4$
$w=3$	$2^w=8$
$x=4$	$2^x=16$
$y=5$	$2^y=32.$

Then $2^y-2^u=32-2=30$ is divisible by 10.

a. What does it mean for a number to be divisible by 10?

b. If N is a number, what are the possible last digits of 2^N?

c. Show that there are two numbers among $2^u, 2^v, 2^w, 2^x, 2^y$ whose difference is divisible by 10.

Last week we learned about *graphs*. Remember that a graph is a collection of dots (called *vertices*) with lines (called *edges*) drawn between some of the vertices.

Problem 3a: Draw a graph with five vertices and four edges.

Problem 3b: Label the vertices in the graph you drew as *1, 2, 3, 4* and *5*.

How many edges are touching vertex *1*?
How many edges are touching vertex *2*?
How many edges are touching vertex *3*?
How many edges are touching vertex *4*?
How many edges are touching vertex *5*?

Problem 3c: Add up the four numbers you wrote in Problem 3b. How is that number related to the number of edges in your graph?

Problem 3d: Anton drew a graph with 6 vertices and 19 edges, but he won't let you see it. Show that one of the vertices in his graph has at least three edges touching it.

Problem 4: There are six people at a party: Alice, Bob, Charlie, Derek, Emily, and Francine. Some people at the party know one another, and some of the people are strangers.

Part A: Represent the people at the party as vertices of a graph. We will draw a red edge between two people if they know one another., and we will draw a blue edge between two people if they are strangers.

Draw a practice graph with red and blue edges.

Is there a red triangle in your graph?
Is there a blue triangle in your graph?

Part B: Draw the vertices of a new graph to represent the people at the party. Show that there are either three people that know Alice or three people who do not know Alice.

Part C: Show that your graph will always have either a red triangle or a blue triangle.

Challenge Problems

1. Five cities in the world are chosen at random. Show that there are **four** cities that all lie on one half of the globe (not necessarily the northern hemisphere or the southern hemisphere though!).

2. There are nine people at a party. Show that there are either four people who are mutual friends or there are three people who are mutual strangers.