Math Challenge

Washington Middle School
January 28, 2010

Geometry

Instructions:
We say that a shape has a *symmetry* if you can make a copy of it, move it around, and fit it back on itself. For example, the letter 'A' has two symmetries: you can either flip it horizontally, or you can just not move it at all.

Exercise 1. How many symmetries do the following letters have? Assume that the letter 'X' is exactly as tall as it is wide.

E:

Q:

C:

N:

X:

H:

Exercise 2. Can you draw a shape with exactly six symmetries? How about a shape with exactly three symmetries? A shape with an infinite number of symmetries?

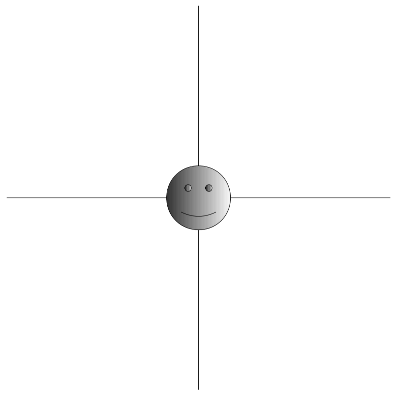
Exercise 3. Let's say your shape is an infinite number of Q's, all in a row, like this:

. . . Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q Q ...

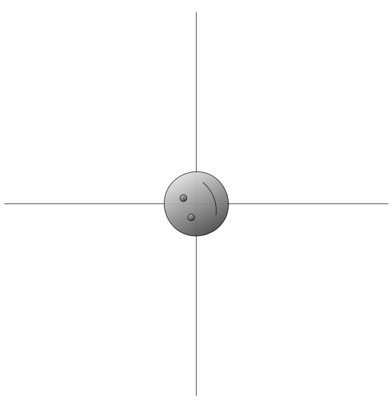
How many symmetries does this have? Can you describe them?

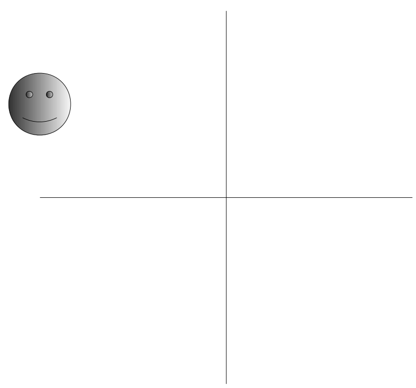
When you move around an object to check if there's a symmetry, the motion is called an *isometry*.

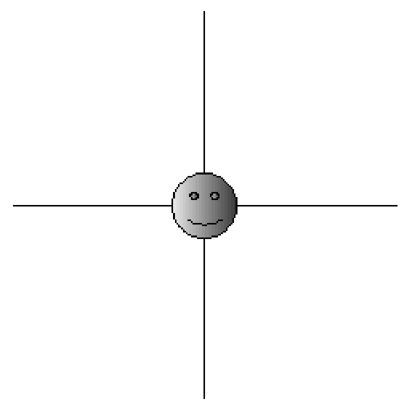
Exercise 4. To help teach math, Steve has decided to become two-dimensional. Here is his head, sitting in the *x*-*y* plane:

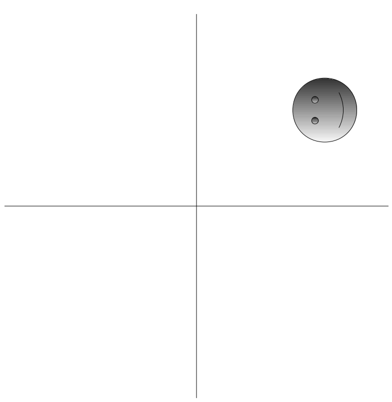


Steve isn't interested in the symmetries of his head, because he doesn't want to end up back where he started (he's a man on the move!). But he is interested in the isometries. Here are some pictures of Steve's head after we move it by an isometry. Can you describe the isometries? Some useful words include *rotation* and *reflection*, but you can describe them any way that is clear. Look carefully at the shading on the heads!





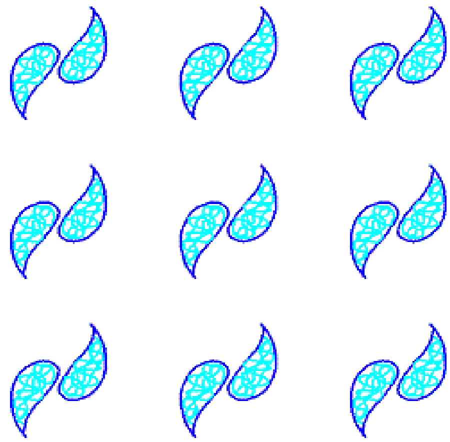




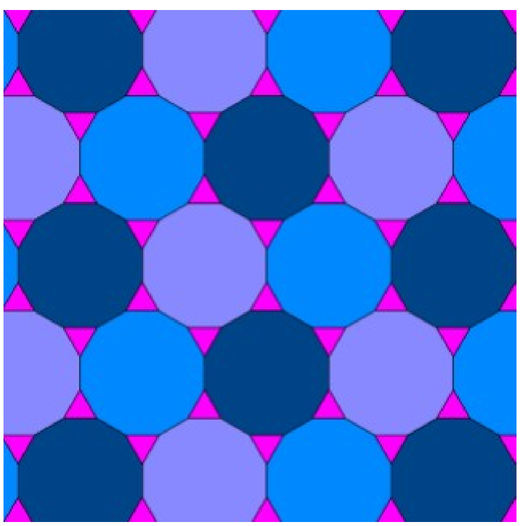
We come to the following scenario:

Exercise 5. A man on the street is selling mathematical facts. “One dollar each! The deal of a lifetime!” he shouts. You go up to see what he's selling, and he tells you that every two-dimensional isometry is either a rotation, reflection, or translation. (You probably already know what a rotation and a reflection are. A *translation* is when you move something without turning or flipping it at all.) Is this “fact” worth a dollar, or is he trying to rip you off? To put it another way, does there exist an isometry that is *not* a rotation, reflection, or translation? If you find one, draw a picture of what it does to Steve's head. If none exist, explain why.

Exercise 6. Imagine that these four designs continue forever. Match up the designs with the descriptions of the symmetries they contain.


Design 1:

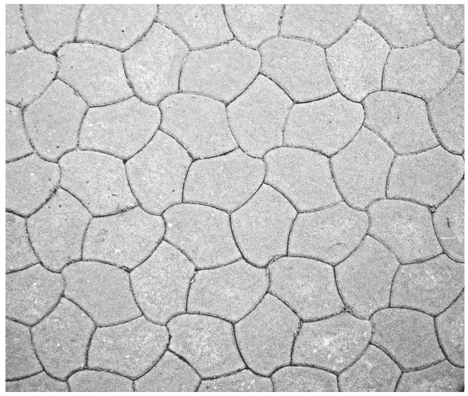

Design 2:


Design 1:

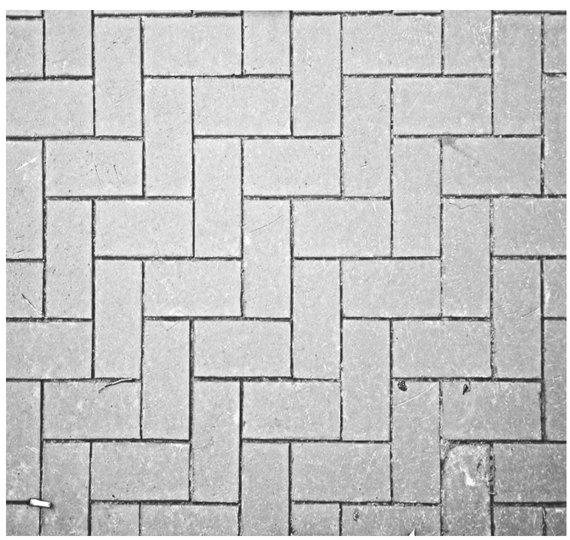

Design 2:

a) rotation by 120 degrees; reflections; translations; and one other type of symmetry (can you find it!)
b) rotation by 60, 120, and 180 degrees; translations
c) translations
d) rotation by 180 degrees; translations

Exercise 7. Here's another picture (it's from a sidewalk in Poland!). What types of symmetries can you find?



Here's another sidewalk. What symmetries can you find?



Exercise 8. Draw some designs of your own on the back of this page! Can you make one with reflections and no rotations? How about 90 degree rotations and no reflections?