Math Challenge

Washington Middle School
February 11, 2010

What is Infinity?

Imagine that you are the manager of a hotel with an infinite number of rooms, numbered 1, 2, 3, ...

Problem 1: It is a dark and stormy night, and every room in your hotel is occupied. Mr. Hilbert arrives at the front desk in the dead of the night, demanding a room. Not wanting to turn away such an esteemed guest, you find space for him in your hotel. How do you do this without making anyone share a room?

Problem 2: After Mr. Hilbert is settled, the basketball team from Infinity Jr. High School arrives. There are infinitely many players on the basketball team, whose jerseys are numbered 1, 2, 3, ... Still, you manage to find a room for every one of the players on the basketball team, again, without making anyone share a room. How?

Problem 3: There is a basketball tournament at Infinity Jr. High this weekend! There will be infinitely many teams playing in the tournament, each with infinitely many players. How can you accommodate everyone in your hotel!?

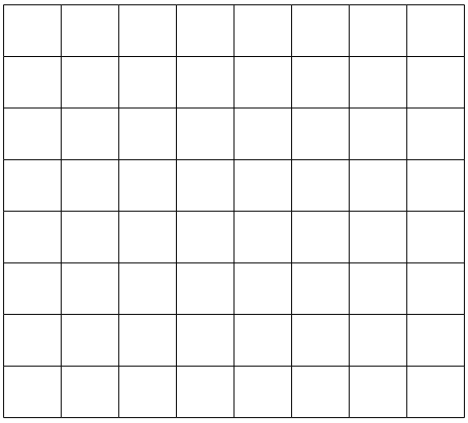
In groups of approximately four people, try to come up with the best solution that you can to each of the following problems. Be sure to justify your answers!

Problem 1: Using the number 8 and the operations of addition, subtraction, multiplication, division, and exponentiation (and parentheses), write a mathematical equation that evaluates to 100. What is the smallest number of 8's that you can use?

For example, a silly solution using 200 8's is the following:

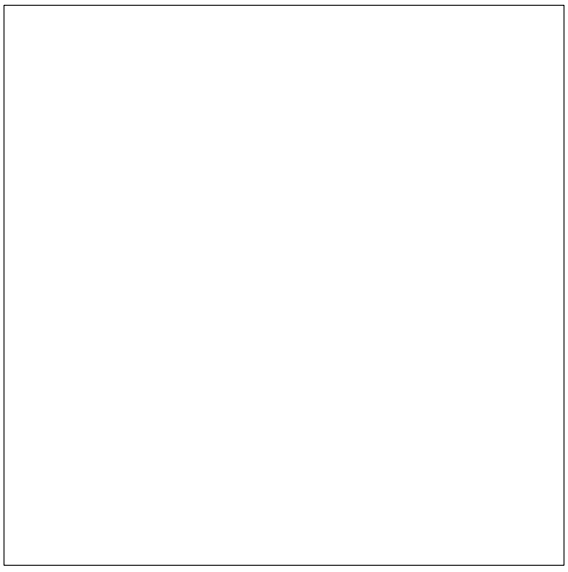
$$(8\div 8) + (8\div 8) + (8\div 8) + \ldots + (8\div 8) \quad (100 \text{ times})$$

Problem 2: What is the largest number of rooks that you can place on an 8x8 chessboard so that each rook attacks an ODD number of other rooks? (Rooks are not transparent – for example, if there are 8 rooks in a single row, then the rook in the first column only attacks one other rook)



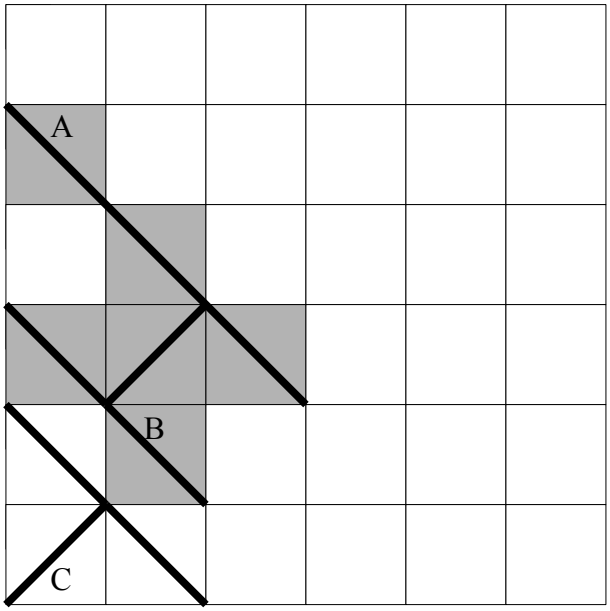
Problem 3: Suppose you are given a block of cheese in the shape of a cube that is 3 inches on each side. You want to cut the block of cheese into 27 cubes of cheese, each of which is 1 inch on each side. Before you make a cut, you can move your pieces of cheese around however you want. What is the smallest number of cuts you can make?

Problem 4: What is the smallest number of acute triangles into which a square can be dissected by straight lines?



Problem 5: In the chessboard drawn below, you will draw one diagonal into each of the 36 squares. A *path* between two squares in the chessboard is a sequence of squares whose diagonals touch. A *component* of the chessboard is a collection of squares, each of which is connected by a path.

For example: In this partial drawing, the shaded squares form a single component. For example, A and B lie in the same component because there is a path of diagonals that connects A and B. The square C does not lie in the same component as A and B.



Question: What if you draw 36 diagonals in the chessboard (one per square), what is the LARGEST number of components you can create?