Math Challenge

Washington Middle School
March 4, 2010

Warm-Up Activity

Pretend that it's **3:00** now. Answer the following questions, but don't worry about AM/PM.

Problem 1a: In 17 hours, what time will the clock show?

Problem 1b: In 33 hours, what time will the clock show?

Problem 1c: What time did the clock show 15 hours ago?

Problem 1d: What time will the clock read 17 hours after the time it shows 19 hours before 4:00?

Today is Thursday. Answer the following questions.

Problem 2a: What day of the week will it be 5 days from now?

Problem 2b: What day of the week will it be 17 days from now?

Problem 2c: What day of the week was it 10 days ago?

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Modular Arithmetic

We use the notation $A = R \ (mod\ N)$ to mean that A has remainder R when divided by N.

Problem 3: Fill in the blanks.

a) 8 (mod 3) = ________

b) 17 (mod 8) = ________

c) 8 (mod 4) = ________

d) 7 (mod 5) = ________

e) 6 (mod 5) = ________

   1. 7 + 6 (mod 5) = ________

   2. 2+1 (mod 5) = ________

f) 20 (mod 11) = ________

g) 35 (mod 11) = ________

   1. 20 + 35 (mod 11) = ________

   2. 9 + 2 (mod 11) = ________

h) 7 (mod 3) = ________

i) 5 (mod 3) = ________

   1. 7 × 5 (mod 3) = ________

j) 5 (mod 4) = ________

   1. 5 × 5 (mod 4) = ________

   2. 5 × 5 × 5 (mod 4) = ________

k) 4 (mod 5) = ________

   1. 4 × 4 (mod 5) = ________

   2. 4 × 4 × 4 (mod 5) = ________

   3. 4 × 4 × 4 × 4 (mod 5) = ________

Problem 4: What is the remainder of $2007 \times 2008 + 2009^2$ when divided by 7?

Problem 5: If your birthday was on a Tuesday last year, on what day will your birthday fall this year? On what day did your birthday fall the previous year?

Problem 6: Pretend you were born on March 2. In 2003, your birthday was on a Monday. On what day did your birthday fall in 2004?

Problem 7: On what day of the week were you born?

Problem 8: Harry goes to the store to buy some candy. He buys:

24 kit-kats
17 peanut butter cups
16 snickers.

Snickers and peanut butter cups cost the same price and the cashier charges Harry $18.65. Harry realizes that the cashier made a mistake and turns him into a toad. How did he know?

Problem 9:* What are the last two digits of 2^{2010}?

Hint: Compute the last two digits of the following numbers:

2^1

2^2

2^4

2^8

2^{16}

2^{32}

2^{64}

2^{128}

2^{256}

2^{512}

2^{1024}

Problem 10: Jeff adds 3 counting numbers $(w + x + y)$ and correctly gets an even sum. Karen adds 2 of the same numbers as Jeff added, plus a different third number $(w + x + z)$ and correctly gets an odd sum. Is the sum of $y + z$ even or odd?

Problem 11*: What is the last digit of the number

$$1^2 + 2^2 + 3^2 + ... + 98^2 + 99^2?$$

Challenge Problems

CP1: Let $N = a_m a_{m-1} \ldots a_2 a_1 a_0$ be an m-digit number with digits $a_0, a_1, \ldots, a_m$.

1. Show that N is divisible by 3 if and only if $a_0 + a_1 + \ldots + a_m$ is divisible by 3.
2. Show that N is divisible by 9 if and only if $a_0 + a_1 + \ldots + a_m$ is divisible by 9.
3. Show that N is divisible by 11 if and only if $a_0 - a_1 + a_2 - \ldots \pm a_m$ is divisible by 11.

CP2: Show that $2222^{5555} + 5555^{2222}$ is divisible by 7.

CP3: Show that $3^{6n} - 2^{6n}$ is divisible by 35 for *any* positive integer n.