HW #3, DUE 1-27

MATH 505A

1. Work out a formula for the number $n(q,p)$ of monic irreducible polynomials of prime degree p over the finite field F_q of order q, by arguing that the minimal polynomial of any element of the larger field F_{q^p} that is not in F_q is one such polynomial and that all arise in this way.

2. Let p_1, p_2 be distinct primes. Extend your reasoning in the previous problem to count the number $n(q, p_1, p_2)$ of monic irreducible polynomials of degree $p_1 p_2$ over F_q.

3. Let α be the positive square root of $(3+\sqrt{3})(2+\sqrt{2})$ in $\mathbb{R}$ and let $K = \mathbb{Q}(\alpha)$ be the field generated by $\mathbb{Q}$ and α. Show that K is also contains $\sqrt{2}$ and $\sqrt{3}$ and is Galois over $\mathbb{Q}$.

4. With notation as in the previous problem, show that the Galois group of K over $\mathbb{Q}$ is isomorphic to the quaternion unit group.

5. Use Sylow theory and the facts from analysis that every polynomial of odd degree over the reals has a real root and every complex number has a complex square root (you need not prove these assertions) to show that the field $\mathbb{C}$ of complex numbers is algebraically closed.