FINAL EXAM

1. Find two examples of a ring R with exactly two prime ideals, one where one of these ideals is contained in the other, the other with neither ideal contained in the other.

2. Classify as completely as you can the fields that are homomorphic images of $\mathbb{Q}[x, y, z]$, the polynomial ring in three variables over $\mathbb{Q}$.

3. Let R be a Noetherian ring. Show that the total ring of quotients $K(R)$ (obtained from R by localizing by all non-zero-divisors) is an Artinian ring.

4. Show that 2 is a square in the ring $\mathbb{Z}_7$ of 7-adic integers.

5. If K is an algebraically closed field, classify the subvarieties of K^n whose coordinate rings are Artinian.

6. Characterize the primary ideals of a Dedekind domain A in terms of the prime ideals of A.

7. If R is a ring, show that the ring $R[x]$ of polynomials in one variable x over R is a faithfully flat extension of R.

8. Let k be an algebraically closed field, A an affine domain over k. We have computed the dimension of A in four different ways (three of them involving a choice of maximal ideal M of A, but all in fact giving the same answer for every M). Describe these ways as clearly as you can (but without giving proofs).