

## Sample Final Problems—Math 506

1. State the version of the Nullstellensatz that holds for arbitrary basefields.
2. Let  $f : A \rightarrow B$  be a homomorphism of rings. Show that the induced map  $f^* : \text{Spec}(B) \rightarrow \text{Spec}(A)$  is continuous with respect to the Zariski topology on both spectra.
3. State the theorem on existence of primary decompositions of ideals in a Noetherian ring, indicating also the two features of this decomposition that are uniquely determined by the ideal.
4. Give an example of a Noetherian domain of dimension one that is not a PID.
5. Give a sufficient condition for a polynomial  $q \in \mathbb{Z}[x]$  to have a solution in  $\mathbb{Z}_p$ , the ring of  $p$ -adic integers, for some prime  $p$ .
6. Show that  $\mathbb{Z}[i]$ , the ring of Gaussian integers, is a faithfully flat extension of  $\mathbb{Z}$ , first by showing that it is flat, and then verifying its faithfulness by showing that the induced map on prime spectra is surjective.
7. Give a necessary and sufficient condition in terms of ideal factorization for one nonzero ideal  $I$  in a Dedekind domain  $A$  to divide another.
8. Show that there is no surjective morphism from  $K^m$  to  $K^n$  for any algebraically closed field  $K$  if  $m < n$ .