HW #4, DUE 4-28

1. Exercise 6.1, Eisenbud, p. 172 (2004 edition).

2. Exercise 6.2, Eisenbud, p. 172.

3. Determine the condition on a ring R for Spec R to be irreducible as a topological space.

4. Give an example of a non-Noetherian ring R such that Spec R is finite.

5. Call a ring R *absolutely flat* if every R-module is flat over R. Show that R is aboslutely flat if and only if every principal ideal (x) in it is idempotent ($(x) = (x^2)$), or if and only if every finitely generated ideal of R is a direct summand of R. (If R is absolutely flat, then show that the flatness of $R/(x)$ implies that the map $(x) \otimes R \to (x) \otimes R/(x)$ is the 0 map, whence $(x) = (x^2)$. Then we have $x = ax^2$ for some a, whence $e = ax$ is idempotent and $(e) = (x)$. If e, f are idempotent elements, then so is $e + f - ef$ and $(e, f) = (e + f - ef)$, whence every finitely generated ideal is principal and generated by an idempotent; finally note that (e) is a direct summand whenever e is idempotent, for then $R = (e) \oplus (1 - e)$.)