# Maximizing Vertical Air on a Quarter Pipe 

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#### Abstract

The current record for maximum vertical air achieved by a snowboarder on a half pipe is 24 feet and 11 inches. We wish to construct such ramps where vertical air is maximized, but the ramp remains executable by a strong snowboarder. We judge the difficulty of ramps by the maximum normal force exerted on the athlete. Limiting that force, allows us to pick executable ramps. We model the snowboarder's trajectory by the his/her center of mass and present two separate models accounting for friction.


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## 1 Introduction

In the halfpipe, snowboarders seek to perform a number of moves. To optimize vertical air we suggest a quarter pipe were incoming velocity and therefore the resulting jump can be higher. Our model seeks the optimal curve to the side of such a snowboard course. Specifically, we analyze the influences of different forces on the center of mass of the boarder during their ride and optimize the curve to maximize their vertical air. First, we model the optimal parameters assuming no friction. Next, we introduce two methods of approximating the influences of friction; one by linearization of the differential equations, the other by numerical integration. We then determine the optimal ellipitic path for the boarder's center of mass, and relate that back to the optimal jump.

## 2 Existing Methods

## FIS Standard Halfpipes

There are two standard FIS halfpipes, each of which is elliptical in shape [3].

1. "22ft" Halfpipe The crown to crown width, which approximates the major axis, is 19.5 meters. The floor to crow height, which approximates the semi-minor axis is $6.5 \mathrm{~m}(22 \mathrm{ft})$. The slope down the half pipe is between $16^{\circ}$ and $17^{\circ}$.
2. " $\mathbf{1 8 f t}$ " Halfpipe The crown to crown width is 18 m meters. The floor to crown height is 5.4 m ( 18 ft ). The slope down the half pipe is between $17.5^{\circ}$ and $18.5^{\circ}$.

The angle at the edges of the pipe are recommended to be between $83^{\circ}$ and $88^{\circ}$

The current world record for vertical air off of a half pipe, held by a skiier, is approximately 25 ft . Based upon our model including friction (), we interpolated backwards and estimated that the skier had a velocity of approximately 16.8 $\mathrm{m} / \mathrm{s}$, which we will use to compare our model with the current model.

## 3 Assumptions

1. Air resistance is negligible. Although strong winds would have a profound influence on the trajectory of a snowboarder, we assume that no wind is present, and that air resistance is negligible by comparison to other frictional forces.
2. Two dimensional solution is similar to three dimensional solution We model the trajectory of the snowboarder in only two dimensions. We assume that an optimal three dimensional surface can be produced by
taking the optimal two dimensional path, and stretching it along the new dimension based upon that component of the velocity.
3. Modeling the center of mass, models the snowboarder. Our model optimizes the path of the center of mass of the snowboarder, which we relate back to the actual shape of the jump.
4. Moment of Inertia is constant. We also assume that the snowboarder's moment of inertia is constant, he approaches the jump standing, and takes off standing.
5. Elite snowboarders are skilled. Our model relies on the ability of the snowboarder to keep proper balance despite the curvature of the jump and that snowboarders' legs can support 4 times their body weight for the length of the jump. (This factor is reasonable based upon Harding [1], but could be easily changed in our model.)
6. Elite snowboarders are well prepared. We use a low snowboardsnow frictionaly coefficient $(\mu=.02)$, based upon the assumption that elite snowboarders wax their boards well, and that the halfpipes are well maintained [2].

## 4 Definitions and Basic Derivations

The following forces act on the snowboarder

- $m g$ : Gravitational Force
- $F_{n}$ : Normal Force
- $F_{f}$ : Frictional Force

Based upon our assumptions, the normal force, $F_{n}$, is constrained by:

$$
F_{n} \leq 4.5 m g
$$

At each point $(x(\theta), y(\theta))$ we have centripetal acceleration in the normal direction $A_{c}$ where

$$
\begin{equation*}
A_{c}=v^{2} \kappa \tag{1}
\end{equation*}
$$

Therefore, adding the normal component of gravitational pull and the normal force we get

$$
\begin{equation*}
m A_{c}=F_{n}-m g \cos (\theta) \tag{2}
\end{equation*}
$$

Ignoring variations in the snowboarder's moment of inertia, we calculate the $F_{n}$ by combining the previous expressions, and noting that $\theta=\arctan \left(\frac{d y}{d x}\right)$ and that $\kappa=\frac{\frac{d^{2} y}{d x^{2}}}{\left(\left(\frac{d y}{d x}\right)^{2}+1\right)^{3 / 2}}$ :

$$
\begin{aligned}
F_{n} & =m v^{2} \kappa+m g \cos (\theta) \\
& =m v^{2} \frac{\frac{d^{2} y}{d x^{2}}}{\left(\left(\frac{d y}{d x}\right)^{2}+1\right)^{3 / 2}}+\frac{m g}{\sqrt{\left(\frac{d y}{d x}\right)^{2}+1}}
\end{aligned}
$$

### 4.1 Goal

The goal of our model is to find the curve that maximizes the vertical air of the snowboarder, given an initial velocity, frictional coefficient and the reactions of the boarder.

We simplify our model from an entire halfpipe to a quarter pipe, and from three dimensions to two. Although each jump depends on the exit velocity of the previous, each jump is isolated from the previous in every other sense. Thus modeling a quarter pipe is very similar to modeling the halfpipe.

In order to maximize the vertical air, we would like to find the point such that the vertical velocity of a snowboarder traveling along the curve is maximal. However, this is unrealistic. As the snowboarder travels along the curve, he or she will experience a great normal force.

## 5 Considerations for Curves

Based upon the discussion above:

1. Slope We want a high final slope, to maximize the snowboarder's vertical velocity.
2. Final Height We want to minimize the final height.
3. Curvature We want a curve that increases in slope quickly, without exerting to great a normal force.

With these considerations in mind, we need to find the types of curves that we can apply standard optimization techniques to.

The curve should reach a steep slope with minimal height, which implies that we would like as high of curvature as possible.

Given that the boarder travels fastest at the bottom, we would like an
increasing curvature, and thus increasing centripetal acceleration, in order to keep the normal force as close to the maximum as possible.

Increasing curvature limits the space of possible solutions significantly. Many common functions, such as monomials, and exponentials, have decreasing curvature, and are thus out of the question. The circle has constant curvature, which makes it much less appealing than it's cousin, the ellipse.

### 5.1 Mathematical formulation

For this model, we will initially assume that there is no friction present in the course. Let $(0,0)$ be the starting position with initial velocity $v_{0}$ and suppose we want the snowboarder to exit the half pipe at the position $\left(x_{f}, y_{f}\right), x, y>0$.

Definition Let $C_{x_{f}, y_{f}}$ denote the circle passing through $(0,0),\left(-x_{f}, y_{f}\right),\left(x_{f}, y_{f}\right)$. Note that such a circle is unique.

The curve the snowboarder will be traveling up will be a section of the lower right quarter of the circle $C_{x_{f}, y_{f}}$. This curve can explicitly be written as

$$
\begin{equation*}
h_{x_{f}, y_{f}}(x)=\frac{x_{f}^{2}+y_{f}^{2}}{2 y_{f}}-\sqrt{\left(\frac{x_{f}^{2}+y_{f}^{2}}{2 y_{f}}\right)^{2}-x^{2}} \tag{3}
\end{equation*}
$$

### 5.2 Results

## 6 Model with Friction (Linearization Method)

In this model, the snowboarder will be traveling up the curve $h_{x_{f}, y_{f}}$ described in the frictionless model, but this time with the effect of friction added. Let $\mu$ be the coefficient of friction. Let $F_{f}(x)$ be the force of friction at position $x$, $0 \leq x \leq x_{f}$. Our model for the force of friction is that

$$
F_{f}(x)=\mu F_{n}(x)
$$

Then as worked out in detail in (), the velocity $v$ satisfies the initial value problem

$$
\begin{aligned}
v^{\prime} & =\mu v \phi^{\prime}-g \frac{\tan \phi+\mu}{v} \\
v(0) & =v_{0}
\end{aligned}
$$



Figure 1: A sample curve $h_{x_{f}, y_{f}}$ for $x_{f}=8.99, y_{f}=6.835, \theta_{f}=88.3$ degrees

| initial velocity | 20 | 20 | 16.8 | 16.8 | 16.8 | 15 | 15 | 15 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| x | 10.000 | 13.000 | 7.999 | 9.000 | 11.998 | 7.000 | 7.999 | 8.998 |
| y | 7.937 | 9.921 | 5.929 | 6.945 | 8.823 | 4.941 | 5.906 | 6.835 |
| launch angle | 89.438 | 89.415 | 89.100 | 89.421 | 88.500 | 89.055 | 88.800 | 88.265 |
| vertical velocity | 15.629 | 14.329 | 12.879 | 12.082 | 10.443 | 11.315 | 10.444 | 9.530 |
| vertical air | 12.449 | 10.465 | 8.454 | 7.440 | 5.558 | 6.525 | 5.560 | 4.629 |

Figure 2: caption

Observe that the above ordinary differential equation is nonlinear. In order to get our results, we will linearize this equation to solve for $v$. Let $\tilde{v}\left(x_{f}\right)$ be the velocity at $x_{f}$ we get from solving the linearized equation. Then the normal force is

$$
\tilde{F}_{n}(x)=\tilde{v}(x)^{2} \kappa\left(h_{x_{f}, y_{f}}\right)+g \frac{1}{\sqrt{h_{x_{f}, y_{f}}^{\prime}\left(x_{f}\right)^{2}+1}}
$$

Then our goal becomes
Maximize the function $\bar{v}\left(x_{f}, y_{f}\right)=\tilde{v}\left(x_{f}\right) \frac{h_{x_{f}, y_{f}}^{\prime}\left(x_{f}\right)}{\sqrt{h_{x_{f}, y_{f}}^{\prime}\left(x_{f}\right)^{2}+1}}$ subject to
the contraints $v_{0}^{2}-2 g y_{f} \geq 0$ and $\max _{0 \leq x \leq x_{f}} \tilde{F}_{N}(x) \leq N_{\max }$

| initial velocity | 20 | 20 | 16.8 | 16.8 | 16.8 | 15 | 15 | 15 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| x | 10.000 | 13.000 | 7.999 | 9.000 | 11.998 | 7.000 | 7.999 | 8.998 |
| y | 7.937 | 9.921 | 5.929 | 6.945 | 8.823 | 4.941 | 5.906 | 6.835 |
| launch angle | 89.438 | 89.415 | 89.100 | 89.421 | 88.500 | 89.055 | 88.800 | 88.265 |
| v velocity | 15.595 | 14.281 | 12.853 | 12.052 | 10.400 | 11.292 | 10.418 | 9.500 |
| v air | 12.395 | 10.394 | 8.419 | 7.403 | 5.512 | 6.499 | 5.532 | 4.600 |

Figure 3: caption

### 6.1 Results

## 6.2 'Optimal' Curve Without Friction

Fixing $F_{n}$ equal to it's bound leads to the following differential equation, which can be derived from the equation for the normal force:

$$
N_{\max }=\sqrt{v_{0}^{2}-2 g y_{f}} * \frac{h^{\prime \prime}(x)}{\left(h^{\prime}(x)^{2}+1\right)^{3 / 2}}+\frac{g}{\left(h^{\prime}(x)^{2}+1\right)^{1 / 2}}
$$

$h(x)$ is optimal in the sense that it will have the minimum final height and maximum curvature for any desired end slope, based upon our limited model. For example, this model ignores friction entirely, which leads to the following model.

## 7 Model with Friction Iterative Method

### 7.1 Description

This model accounts for energy lost due to friction, by using numerical approximations for friction along points of the curve.

Our goal is to calculate the ending velocity by considering the total non conservative energy lost in the ride up the slope.

### 7.2 Mathematical Formulation

The snowboarder is treated as a particle of mass $m$ traveling along the curve $(x(\theta), y(\theta))$ for $\theta_{s}<\theta<\theta_{f}$. At each point we have

$$
\begin{aligned}
& F_{n}(\theta)=m * v^{2}(\theta) * \kappa(\theta)+m g * \cos (\phi(\theta)) \\
& F_{f}(\theta)=\mu F_{n}
\end{aligned}
$$

where

$$
\begin{aligned}
\kappa(\theta) & =\frac{x^{\prime} y^{\prime \prime}-y^{\prime} x^{\prime \prime}}{\left(x^{\prime 2}+y^{\prime 2}\right)^{3 / 2}} \\
\phi(\theta) & =\arctan \left(\frac{y^{\prime}(\theta)}{x^{\prime}(\theta)}\right)
\end{aligned}
$$

Taking a partition $\left[\theta_{s}, \theta_{1}, \ldots, \theta_{f}\right]$ we can calculate $v\left(\theta_{n}\right)$ by considering the non conservative energy $W_{n c}$ lost in the interval $\left[\theta_{n-1}, \theta_{n}\right]$. The only non conservative force we consider here is friction.

$$
W_{n c}=\int_{\theta_{n-1}}^{\theta_{n}} F_{f}(\theta) d s(\theta)
$$

For small enough $\theta_{n-1}-\theta_{n}$, we can approximate $F_{f}(\theta)$ for $\theta_{n-1} \leq \theta \leq \theta_{n}$ by $F_{f}\left(\theta_{n-1}\right)$. We get

$$
\begin{aligned}
W_{n c} & \approx F_{f}\left(\theta_{n-1}\right) \int_{\theta_{n-1}}^{\theta_{n}} d s(\theta) \\
W_{n c} & \approx F_{f}\left(\theta_{n-1}\right) \sqrt{\left(y\left(\theta_{n-1}\right)-y\left(\theta_{n}\right)\right)^{2}+\left(x\left(\theta_{n-1}\right)-x\left(\theta_{n}\right)\right)^{2}}
\end{aligned}
$$

We can now calculate $v\left(\theta_{n}\right)$ by the work-energy theorem

$$
\frac{1}{2} m v^{2}\left(\theta_{n}\right)=\frac{1}{2} m v^{2}\left(\theta_{n-1}\right)+m g * y\left(\theta_{n-1}\right)-m g * y\left(\theta_{n}\right)-W_{n c}
$$

Therefore given $v\left(\theta_{s}\right)$, we can calculate $F_{f}\left(\theta_{s}\right), v\left(\theta_{s+1}\right)$ and so on up to $v\left(\theta_{f}\right)$

### 7.3 Data

We considered ellipses $x=a \cos (\theta), y=b \cos (\theta)$ for $-\frac{\pi}{2} \leq \theta \leq 0$.
The estimations bellow are taken from running the algorithm presented above dividing the interval for $\theta$ into 200 equal sections. When selected calculations were run with 10000 sub-sections the results remained the same to 3 significant figures.

In our analysis we varied the initial velocity at the base of the quarter pipe, as well as the shape of the ellipse. For each three-tuple of conditions we calculated the vertical air the boarder would achieve as well as the max normal force they experienced at any sampled point. The consideration of the max normal force on the boarder allowed us to rule out designs, even if they would theoretically lead to higher vertical air.

Effects of changing the frictional coefficient were also examined. However the data is not presented. Although the friction did somewhat change the vertical air achieved, it had significantly less effect than initial velocity on the max normal force experienced, and therefore on the reasonableness of the design.

Instead we present data to support two very key points:

1. Increasing initial speed, although resulting in higher vertical air, comes at a very large cost of increased normal force on the athlete.
2. More precise data about boarder ability to withstand normal force is necessary. There is a very large variation between curves in how much normal force they will exert. Therefore a better estimate on the upper bound of human strength will allow us to design a resaonable quarter pipe to maximize vertical air.

Consider the tables bellow for the max normal force and vertical air achieved for selected ellipses with an initial velocity of $16.8 \mathrm{~m} / \mathrm{s}$. The cells are color coded to represent the strength of maximum normal force. Given a range of max normal forces our recommendation for a quarter pipe design is bolded.

|  | "g's" (Force normal divided by mg, with initial velocity $16.8 \mathrm{~m} / \mathrm{s}$ ) Varying Length Horizontal Axis (m) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|  | 5 | 5.43 | 5.93 | 6.51 | 7.12 | 7.75 | 8.39 | 9 | 9.67 | 10.31 |
| $\times$ | 6 | 4.53 | 4.28 | 4.46 | 4.74 | 5.07 | 5.41 | 5.78 | 6.15 | 6.52 |
| Tow | 7 | 5.3 | 4.15 | 3.65 | 3.61 | 3.7 | 3.85 | 4.04 | 4.24 | 4.45 |
| $\stackrel{\stackrel{\rightharpoonup}{\nu}}{>}$ | 8 | 5.69 | 4.59 | 3.84 | 3.3 | 3.12 | 3.09 | 3.13 | 3.2 | 3.3 |
| 5 | 9 | 6.28 | 5.04 | 4.2 | 3.59 | 3.14 | 2.84 | 2.73 | 2.69 | 2.69 |
| ¢ | 10 | 6.86 | 5.4 | 4.55 | 3.88 | 3.38 | 3 | 2.7 | 2.52 | 2.43 |
| 0 | 11 | 7.45 | 5.94 | 4.9 | 4.16 | 3.61 | 3.2 | 2.87 | 2.61 | 2.41 |
| 2 | 12 | 8.04 | 6.39 | 5.25 | 4.45 | 3.85 | 3.4 | 3.04 | 2.76 | 2.53 |
| g.png ${ }^{\text {¢ }}$ | 13 | 8.63 | 6.84 | 5.61 | 4.74 | 4.09 | 3.6 | 3.21 | 2.91 | 2.66 |

"g's" (Force normal divided by mg, with initial velocity $16.8 \mathrm{~m} / \mathrm{s}$ )
Varying Length Horizontal Axis (m)

| $\underset{\frac{n}{x}}{\mathbb{E}}$ |  | 78 |  | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5 | 5.43 | 5.93 | 6.51 | 7.12 | 7.75 | 8.39 | 9 | 9.67 | 10.31 |
|  | 6 | 4.53 | 4.28 | 4.46 | 4.74 | 5.07 | 5.41 | 5.78 | 6.15 | 6.52 |
| TV | 7 | 5.3 | 4.15 | 3.65 | 3.61 | 3.7 | 3.85 | 4.04 | 4.24 | 4.45 |
| $\stackrel{\otimes}{\otimes}$ | 8 | 5.69 | 4.59 | 3.84 | 3.3 | 3.12 | 3.09 | 3.13 | 3.2 | 3.3 |
| 5 | 9 | 6.28 | 5.04 | 4.2 | 3.59 | 3.14 | 2.84 | 2.73 | 2.69 | 2.69 |
| $\stackrel{\Gamma}{0}$ | 10 | 6.86 | 5.4 | 4.55 | 3.88 | 3.38 | 3 | 2.7 | 2.52 | 2.43 |
| 1 | 11 | 7.45 | 5.94 | 4.9 | 4.16 | 3.61 | 3.2 | 2.87 | 2.61 | 2.41 |
| $\frac{5}{\Sigma}$ | 12 | 8.04 | 6.39 | 5.25 | 4.45 | 3.85 | 3.4 | 3.04 | 2.76 | 2.53 |
| 5 | 13 | 8.63 | 6.84 | 5.61 | 4.74 | 4.09 | 3.6 | 3.21 | 2.91 | 2.66 |

If the human can only support less than $3 m g$ of normal force, the best quarter pipe design would be an ellipse with $a=12, b=9$. The boarder would then achieve about 4.52 meters of vertical air.

However if the human strength does allow for 5 mg of normal force, then the best pipe design would be $a=8, b=6$. On this design, vertical air achieved would be much higher at 7.51 meters

Data for initial velocities of $15 \mathrm{~m} / \mathrm{s}$ and $20 \mathrm{~m} / \mathrm{s}$ is included in the appendix, and similar conclusions can be drawn.

### 7.4 Model improvements

Previously, we were considering the full quarter ellipse with $\frac{-\pi}{2} \leq \theta \leq 0$. However the ramp exit need not necessarily be vertical.

We have been modeling boarders as their center of mass, but as they leave the ramp, boarders are upright, and coming down they bend in their knees. This is about a $\frac{1}{2}$ meter difference in the center of mass. Which means their center of mass is allowed that much horizontal travel, while still letting the boarder land back on the ramp.

Therefore, we modified our code to find the best point to truncate the ramp that allows for a proper landing. The data for a sample ellipse is bellow

| a | 8 | 9 | 12 |
| :--- | ---: | ---: | ---: |
| b | 6 | 7 | 9 |
| launch angle with cutoff | 89.1 | 89.42 | 88.5 |
| vertical air with cutoff | 7.5873 | 6.57407 | 4.6939 |
| vertical air without cutoff | 7.51 | 6.52 | 4.52 |
| Gain in vertical air | 0.0773 | 0.05407 | 0.1739 |

## 8 Results of Model 2

## 9 Other Considerations and Limitations

### 9.1 Moment of Inertia

In our model we assumed that the moment of inertia of the boarder was constant, and that the boarder did not change his own center of mass. These assumptions entirely ignore two influential factors in the snowboarder's jump.

### 9.1.1 Pumping

Pumping is a way for snowboarders to increase the velocity of their center of mass by decreasing their moment of inertia while in a curve. The velocity increase comes from conservation of angular momentum. If the boarder enters the curve crouched, and stands up as he reaches the top, he will increase his speed, and thus his vertical air.

### 9.1.2 Popping Off

At the top of the curve, the snowboarder can suddenly push off, which helps cancel out any horizontal velocity that he still has. As such, the jump can be made shorter, and less steep at the top, thus increasing vertical air.

### 9.2 Movement in Air

Although vertical height is important, it is not the overall end goal of a snowboarder. Tricks, such as grabs, spins, and flips are what snowboarders strive for. In order to model such tricks, a three dimensional model would have been necessary.
Also, vertical height is just a proxy for something more important, average air time. Average air time is what allows snowboarders to perform more difficult tricks in the air, and it is strongly correlated with success in competitions.

## 10 Conclusions

We conclude that the standard snowboard halfpipe leaves something to be desired when seeking vertical air.

Consider the table bellow summarizing our three models for the initial velocity of $16.8 \mathrm{~m} / \mathrm{s}$. In all cases the vertical air is greater than 24 feet 11 inches. We

|  | no friction | Iterative friction | Linearization friction |
| :--- | ---: | ---: | ---: |
| vertical velocity | 12.88 | 12.2 | 12.85 |
| vertical air | 8.45 | 7.59 | 8.42 |

find that the initial velocity has a profound influence on the normal forces that the snowboarder experiences in the jump, and that in order to achieve more vertical air, an ellipse with less curvature should be chosen.

## 11 Appendix

|  | Vertical Air (Initial velocity $15 \mathrm{~m} / \mathrm{s}$ ) <br> Varying Length Horizontal Axis (m) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underline{E}$ |  | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| $\stackrel{4}{x}$ | 5 | 5.77 | 5.76 | 5.75 | 5.74 | 5.73 | 5.71 | 5.7 | 5.69 | 5.67 |
| $\stackrel{5}{9}$ | 6 | 4.78 | 4.77 | 4.76 | 4.75 | 4.74 | 4.73 | 4.7 | 4.7 | 4.69 |
| $\stackrel{5}{v}$ | 7 | 3.79 | 3.78 | 3.777 | 3.77 | 3.78 | 3.748 | 3.74 | 3.72 | 3.71 |
| 8 | 8 | 2.78 | 2.8 | 2.79 | 2.78 | 2.77 | 2.76 | 2.76 | 2.74 | 2.73 |
| 들 | 9 | 1.81 | 1.8 | 1.8 | 1.79 | 1.78 | 1.77 | 1.77 | 1.76 | 1.75 |
| air.png ${ }^{5}$ | 10 | 0.81 | 0.81 | 0.8 | 0.8 | 0.79 | 0.78 | 0.78 | 0.77 | 0.76 |

f.jpg


\footnotetext{
Vertical Air (Initial velocity 20m/s)

|  |  | 8 |  | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 7 | 12.17 | 12.16 | 12.16 | 12.15 | 12.1 | 12.13 | 12.12 | 12.1 | 12.09 |
|  | 8 | 11.17 | 11.17 | 11.16 | 11.16 | 11.15 | 11.14 | 11.13 | 11.12 | 11.11 |
|  | 9 | 10.18 | 10.18 | 10.17 | 10.17 | 10.16 | 10.15 | 10.14 | 10.13 | 10.12 |
|  | 10 | 9.18 | 9.18 | 9.18 | 9.1 | 9.17 | 9.16 | 9.16 | 9.15 | 9.14 |
|  | 11 | 8.19 | 8.19 | 8.19 | 8.18 | 8.18 | 8.17 | 8.17 | 8.16 | 8.15 |
|  | 12 | 7.19 | 7.19 | 7.19 | 7.19 | 7.19 | 7.18 | 7.18 | 7.17 | 7.16 |
|  | 13 | 6.2 | 6.2 | 6.2 | 6.2 | 6.19 | 6.19 | 6.18 | 6.18 | 6.17 |
|  | 14 | 5.2 | 5.2 | 5.2 | 5.2 | 5.2 | 5.2 | 5.19 | 5.18 | 5.18 |
|  | 15 | 4.2 | 4.2 | 4.2 | 4.2 | 4.2 | 4.2 | 4.2 | 4.2 | 4.19 |



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