When Weather Warms, Water Wises

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Abstract

Scientific research concerned with modeling global climate began to emerge during the 1960s. The research suggests that human generation of greenhouse gases such as carbon dioxide and methane is producing sharply rising temperatures. In the next century, the most important result of this global warming will be the melting of the polar ice caps and the associated rise in sea levels. We develop a model for global climate change, with both latitudinal and longitudinal resolution. Our model is derived by applying the principle of energy conservation to each element of the discretized surface of the Earth. We couple this model with a model for melting rates and snow accumulation at the polar ice caps, and apply these models to determine sea levels over the next five decades. As a case study, we apply our energy-balance model to predict the distribution of the population of the state of Florida that will be displaced by the year 2057, and we determine how these results will change if the world's greenhouse gas production is raised or lowered. We conclude that changing greenhouse gas production will not significantly affect the number of Floridians displaced over the next half-century.

Contents

1 Introduction

To quote the president of London's Royal Society in 1817:

> It will without doubt have come to your Lordship's knowledge that a considerable change of climate, inexplicable at present to us, must have taken place in the Circumpolar Regions, by which the severity of the cold that has for centuries past enclosed the seas in the high northern latitudes in an impenetrable barrier of ice has been during the last two years, greatly abated.
>
> (This) affords ample proof that new sources of warmth have been opened and give us leave to hope that the Arctic Seas may at this time be more accessible than they have been for centuries past, and that discoveries may now be made in them not only interesting to the advancement of science but also to the future intercourse of mankind and the commerce of distant nations [15].

Though the scientific community has by now discovered better methods for understanding the atmosphere than this President had in 1817, global climate modeling remains an elusive task. This is due simply to the incredibly complex, interconnected nature of the Earth's climate system: heat is transferred in three distinct ways, convection, conduction, and radiation; this transfer differs depending on the medium, either sea-water, rock crust, or atmospheric gas; each of these media need to be divided into a certain number of layers in order for heat flow to be modeled in an accurate fashion; and large amounts of so-called *greenhouse-gases* are constantly introduced by humans. Nearly every scientist can agree that man began to produce large amounts of greenhouse gases during the Industrial Revolution, and that these gases have led to an overall warming trend. The exact effects of this trend are much more difficult to determine.

As surface temperatures rise, an extremely pressing question is,

- Will global warming melt enough of the polar ice caps to threaten the lives of people dwelling in low-lying areas?

The fact that more than half of the world's population lives within 120 miles of a seacoast, which corresponds to roughly 10% of total landmass, highlights the urgency of answering this question [**?**]. In this paper, we will develop a model for the rate of rise of average sea level, and use this model to determine whether the residents of the state of Florida are in danger of massive flooding sometime during the next 50 years. The model will be based on radiative forcing due to the presence of greenhouse gases, and heat transfer between the atmosphere-ocean-land system and the North polar ice cap.

2 Plan of attack

Our goal is to determine the effects, adverse and otherwise, on Florida as a result of global warming. We could focus on many things — changing current patterns, changing water temperature and thus changing hurricane patterns, changing precipitation patterns, or changing salinity levels, to name a few — but we choose to limit ourselves to the central result: rising sea levels. To model global sea levels, we must:

- Formulate a climate model, in sufficient generality that the rates of melting of the Earth's ice sheets can be easily derived.

- Apply this model to find the total amount of runoff water from melted ice sheets in the years 2017, 2027, 2037, 2047, and 2057.
- Use the runoff data to determine the portions of Florida that will be underwater in the years 2017, 2027, 2037, 2047, and 2057.

3 Simplifying assumptions

3.1 Contributions to sea-level variation

Variation in sea-level is caused in two ways:

- Ice melts, and runs off into the ocean. Or,
- The sea warms and its water undergoes thermal expansion.

Ice run-off contributes roughly 80–90% of the total sea-level rise [21]. Furthermore, the simplest realistic models for thermal expansion involve partitioning the ocean into many layers, and modeling heat transmission with both standard diffusion and a diffusion-convection hybrid called upwelling-diffusion. Simulating these dynamics is complicated, and we will not attempt a simulation here. Since thermal expansion is a relatively small component of the total sea-level variation, we make our first simplification:

- The level of the sea is dependent only on run-off from melting ice-sheets.

3.2 Ice sheets

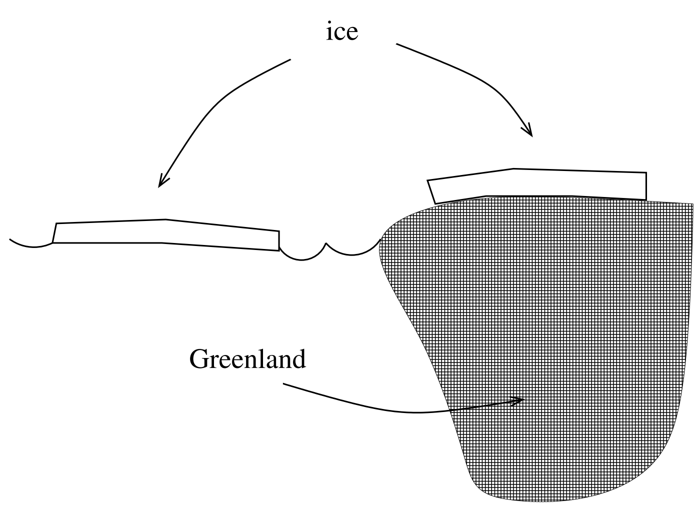

Figure 1: An illustration of the two types of ice in the North polar ice cap.

One need only glance at Figure 4 to see that the South pole is *much* colder than the North pole. Indeed, while it is widely accepted that the Greenland ice sheet is melting [14], [8], [20], many authors disagree on whether the Antarctic ice sheet is melting or not: while [9] suggests that the Antarctic sheet is losing mass to the ocean, [8] suggests that Antarctica is actually gaining mass! It would be far too difficult to attempt to simulate the dynamics of the Antarctica sheet, and so we make the second simplifying assumption:

- Antarctica is neither losing nor gaining mass.

After making this assumption, we might assume that the contribution to rising sea levels by melting ice comes from the entire North polar ice cap. However, this is *not* the case. Consider two pieces of ice in the North polar cap, one floating on the ocean, and one fixed atop Greenland (see Figure 1). If the piece atop Greenland melts, it will run off into the sea, Greenland will remain where it was, and the ocean will rise. *However*, the floating chunk of ice displaces its own weight in water, and if it melts, it will continue to displace this same amount of water. That is, if the floating piece melts, sea-level will not change. This argument shows that sea-level is affected only by the melting of ice-sheets that lie on top of fixed land-masses. As stated in [14], the vast majority of non-floating ice is contained in either the Antarctic ice sheet or the Greenland ice sheet. We can now safely assume that

- Greenland is the sole contributor to the variation in sea-level.

4 The energy-balance model

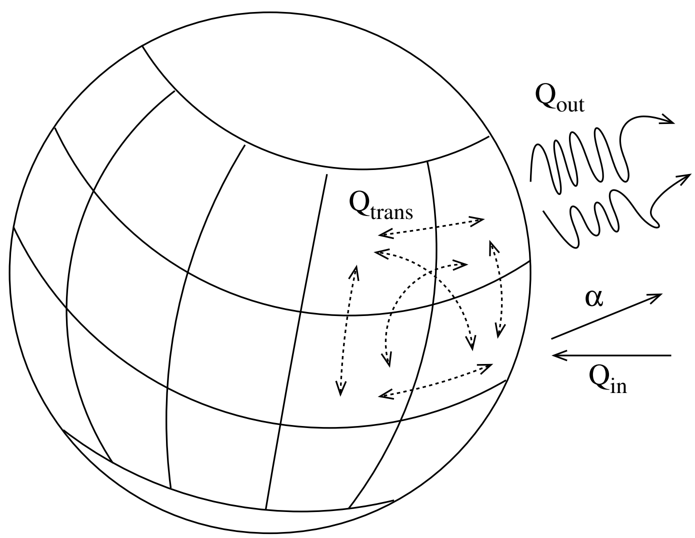

Figure 2: A schematic of the two-dimensional energy-balance model.

The simplest method of modeling global surface temperature is to partition the Earth into a (possibly infinite) number of compartments, and to simulate energy transfer between these compartments by enforcing the conservation of energy. These models were the first to realistically predict long-term climate patterns, and many of the modern climate models have an energy-balance model at their core. A number of simplifying assumptions must first be made:

- In every compartment, temperature is uniform.
- Heat is transferred only through radiation and conduction, and not through convection.
- When heat diffuses from the Earth's surface into the Earth, this heat immediately distributes itself uniformly.

Energy-balance models are governed by a single partial differential equation of the form

$$R \cdot \frac{\partial T}{\partial t}(\mathbf{x}) = \left.\frac{\partial Q}{\partial t}\right|_{\text{in}}(\mathbf{x}) - \left.\frac{\partial Q}{\partial t}\right|_{\text{out}}(\mathbf{x}, T) - \left.\frac{\partial Q}{\partial t}\right|_{\text{trans}}(\mathbf{x}, T). \tag{1}$$

The constant R is the thermal capacity of the atmosphere, so that the left side of (1) is equal to the rate of change of the thermal energy of each compartment. Following Tung [19], we will take $R = 0.85 \cdot \text{J} \cdot {}^\circ\text{C}^{-1} \cdot \text{m}^{-2}$. As for the right side of the equation, we note that

- $\partial Q/\partial t|_{\text{in}}$ denotes solar radiation per square meter. This is only a function of position, since the temperature of a compartment does not affect its absorption of solar energy.
- $\partial Q/\partial t|_{\text{out}}$ denotes the rate of thermal energy per square meter radiated back into space. This is a function of *both* temperature and position.
- $\partial Q/\partial t|_{\text{trans}}$ denotes the rate of transfer of thermal energy per square meter from one compartment to another. By convention, flow from a compartment into its neighbors is positive, while flow into a compartment from its neighbors is negative.

4.1 Types of energy-balance models

The central difference between different energy-balance models is the choice of the partitioning of the Earth into compartments. In practice [11], there are three types of partitions.

- The first type of partition is to consider the entire surface of the Earth to be a single compartment.
- The second is to partition based on latitude, so that each compartment is a single latitudinal band.
- The third is to partition based on both latitude and longitude, so that each compartment is a square-like quadrilateral on the Earth's surface. (See Figure 2 for a schematic illustration of a model that uses this type of partition.)

All energy-balance models are based on the same principle of thermal bookkeeping. How reasonable the model is depends strongly on the type of partition. If we use a partition of the first type, where there is only one compartment, all our model will predict is a global average temperature. While this may be useful for predicting temperature trends on extremely long time-scales, it is not at all useful for predicting the melting speed of Greenland. If we equated the surface temperature of Greenland with the average global temperature, the rate of melting would be grossly exaggerated.

Both the second and third types of partition produce reasonable predictions [13]. For our model, we will partition both latitudinally and longitudinally.

4.2 Idealized model

First, we will describe an energy-balance model that does *not* take into consideration the melting of ice. Whether or not our model incorporates melting is important, since ice absorbs a certain amount of energy when it makes the phase transition from solid to liquid. For now, the governing equation for our model is

$$R \cdot \frac{\partial T}{\partial t}(\mathbf{x}) = S(\mathbf{x}) \cdot (1 - \alpha(\mathbf{x})) - (A + B \cdot T(\mathbf{x})) - K \cdot \left(T(\mathbf{x}) - \overline{T}(\mathbf{x})\right). \tag{2}$$

In this equation,

- $S(\mathbf{x})$ is insolation, which is the latitude-dependent irradiation of the Earth by the Sun. S is gotten empirically; its source is described later in the paper.
- $\alpha(\mathbf{x})$ is the albedo, which is the proportion of solar radiation that is reflected back into space. α is also gotten empirically; its source is also described later on.
- $A + B \cdot T(\mathbf{x})$ is the linearized form of the radiation back into space. The constant A and B vary dramatically depending on the source of measurements. We choose the reasonable values $A = 204 \cdot \mathrm{W} \cdot \mathrm{m}^{-2}$ and $B = 2.6 \cdot \mathrm{W}^{\circ}\mathrm{C}^{-1} \cdot \mathrm{m}^{-2}$.
- K is an empirical constant, chosen so that $K\left(T(\mathbf{x}) - \overline{T}(\mathbf{x})\right)$ approximates the rate of diffusion of thermal energy. Following [7], we choose $K = 3.8 \cdot \mathrm{W} \cdot \mathrm{m}^{-2} \cdot {}^{\circ}\mathrm{C}^{-1}$.

4.3 Introducing time-dependent melting and accumulation

The central problem of this paper is determining the melt rate of Greenland. Naïvely, we could use the model described in the last section to predict temperature as a function of both t and $\mathbf{x}$, and then use this temperature data to predict how quickly Greenland will melt over the next 50 years. However, as Greenland melts, along with the rest of the North polar icecap, some thermal energy is transferred to induce the phase transition of ice into water. To accurately model temperature, we must incorporate an ice-to-water phase-transition term into equation (2).

Polar ice melts only at temperatures above 12°C [14]. Furthermore, since ice typically does not exist in regions which regularly experience temperatures above 0°C, we need only consider the melting of ice sheets with mean annual temperature between 0°C and 12°C. For these regions, the empirically-determined melting rate, again taken from [14], is given by

$$\left.\frac{\partial Q}{\partial t}\right|_{\text{melt}}(\mathbf{x}, T) = \Delta_h \left(\cdot M \cdot (12^{\circ}\mathrm{C} + T)^2 - R_m(\mathbf{x})\right),$$

where $M = 8.13 \cdot 10^{-7}$ kg $\cdot$ $^{\circ}\mathrm{C}^{-2} \cdot \mathrm{m}^{-2} \cdot \mathrm{s}^{-1}$, Δ_h is the heat of fusion of water, and $m(\mathbf{x}) = \max\left(0.05, \left(0.3 + 14s - 4 \cdot 10^{-5}h\right)/C\right)$, where s is slope, h is height, and $C = 1.32$.. After introducing this term, equation (2) becomes

$$R \cdot \frac{\partial T}{\partial t}(\mathbf{x}) = S(\mathbf{x}) \cdot (1 - \alpha(\mathbf{x})) - (A + B \cdot T(\mathbf{x})) - K \cdot \left(T(\mathbf{x}) - \overline{T}(\mathbf{x})\right) - \Delta_h \left(\cdot M \cdot (12^{\circ}\mathrm{C} + T)^2 - R_m(\mathbf{x})\right). \tag{3}$$

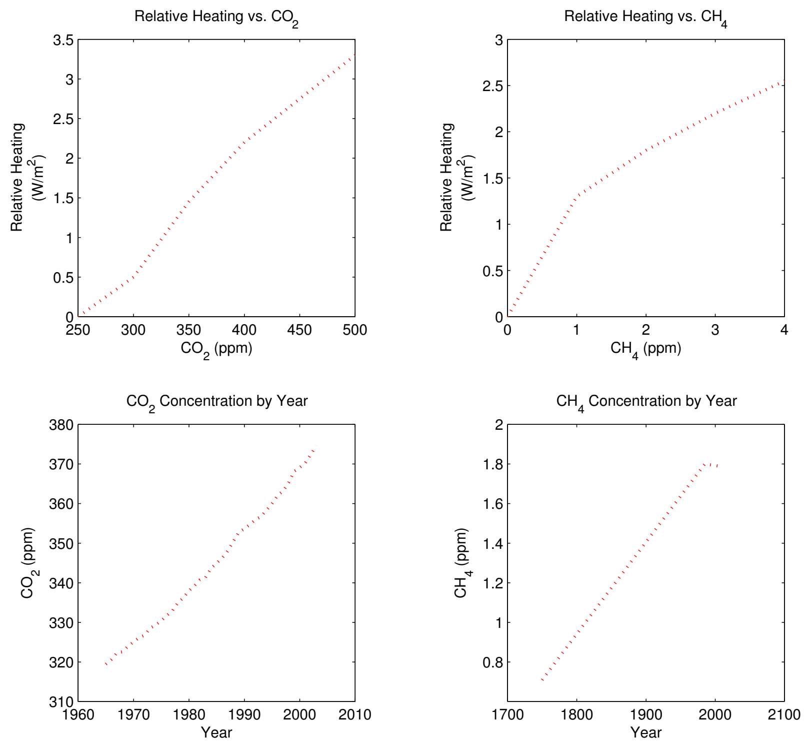

Figure 3: Experimental data of both radiative heating versus carbon dioxide and versus methane levels, and of carbon dioxide and methane levels versus year.

4.4 Introducing increased radiation due to greenhouse gases

Equation (3) is valid as long as atmospheric conditions do not deviate from the conditions at the time of the empirical derivation of A and B. However, it is seldom the case that the chemical composition of the atmosphere remains constant: trace gases such as carbon dioxide, methane, nitrous oxide, and chlorofluorocarbons are in constant flux. For this reason, we must add a final term to the governing equation, due to the varying concentrations of trace greenhouse gases.

We will use the empirical data of increased radiation per square meter versus concentrations of greenhouse gases given in [4], and the concentrations of greenhouse gases versus year taken from [1]. See Figure 3 for the data for carbon dioxide and methane, which are by far the most significant greenhouse gases.

With this data in hand, we can add an term corresponding to greenhouse gases to equation (3), in order to arrive at our final governing equation:

$$R \cdot \frac{\partial T}{\partial t}(\mathbf{x}) = \left.\frac{\partial Q}{\partial t}\right|_{\text{in}}(\mathbf{x}) - \left.\frac{\partial Q}{\partial t}\right|_{\text{out}}(\mathbf{x}, T) - \left.\frac{\partial Q}{\partial t}\right|_{\text{trans}}(\mathbf{x}, T) - \left.\frac{\partial Q}{\partial t}\right|_{\text{melt}}(\mathbf{x}, T) + \left.\frac{\partial Q}{\partial t}\right|_{\text{gas}},$$

for

$$\begin{cases} \partial_t Q|_{\text{in}}(\mathbf{x}) & = S(\mathbf{x}) \cdot (1 - \alpha(\mathbf{x})) \\ \partial_t Q|_{\text{out}}(\mathbf{x}) & = (A + B \cdot T(\mathbf{x})) \\ \partial_t Q|_{\text{trans}}(\mathbf{x}) & = K\left(T(\mathbf{x}) - \overline{T}(\mathbf{x})\right) \\ \partial_t Q|_{\text{melt}}(\mathbf{x}) & = \Delta_h \left(\cdot M \cdot (12°\text{C} + T)^2 - R_m(\mathbf{x})\right) \\ \partial_t Q|_{\text{gas}} & = G(t) \end{cases} . \tag{4}$$

Here the function G is linearly extrapolated from the empirical data described above.

5 Data

Unfortunately, accurate and complete data sets relevant to climate change are very difficult to find. We have tried to be as careful and complete with these data as possible, and in this section we will give a summary of our data sets along with brief discussions of their reliability.

5.1 Temperature data

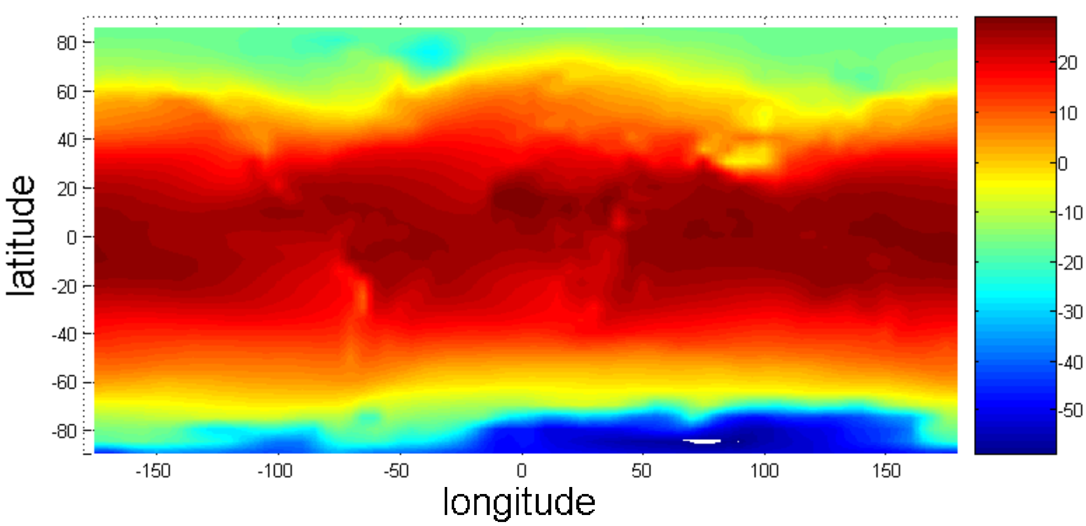

Figure 4: An interpolated plot of temperature measurements taken in 1990.

Temperature datasets are produced by a joint effort of the Met Office Hadley Centre and the Climatic Research Unit at the University of East Anglia. There is a dataset for each year from 1850 until 2007, and each dataset consists of a $5° \times 5°$ grid of interpolated temperature data point, in degrees Celsius. The land data is gotten from the independent measurements of 3000 observing stations distributed across the globe, while the sea data has been taken by volunteer naval vessels. These data are not disputed, and careful analyses of the methodology of these data sets can be found in [10] and [16]. See Figure 4 for a plot of these data.

5.2 Population data

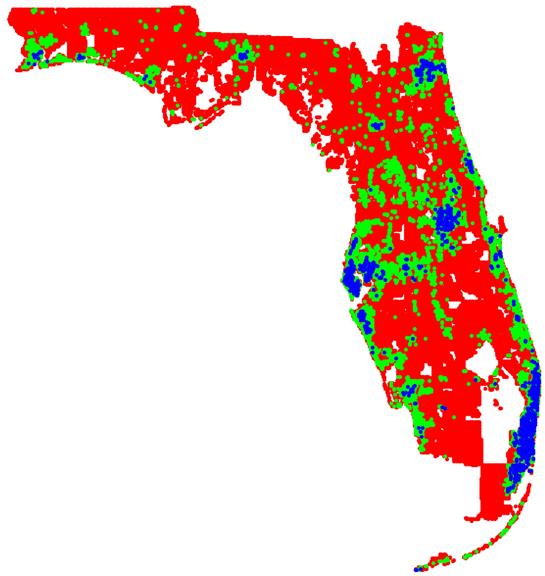

Figure 5: A plot of the population density of Florida, as of 2000. Red dots correspond to census tracts with fewer than 100 people; green dots correspond to census tracts with between 100 and 1000 people; and blue dots correspond to census tracts with more than 1000 people.

Population datasets are taken from the 2000 U.S. census, as stored by Columbia University's Socioeconomic Data and Applications Center. Each data-point represents the number of people living in a certain census tract. There is essentially no question that these data are accurate. See Figure 5 for a plot of some of these data.

5.3 Topographical data

Topography datasets are taken from the National Geophysical Data Center's `ETOPO2` 2″ grid [5]. The data set is sampled so that a more manageable 10″ grid is gotten. See Figure 6 for a plot of these data. It is apparent from this plot that much of Florida's southern regions lie at very, very low elevations. There is essentially no question that these data are accurate.

5.4 Greenhouse gas data

Empirical datasets both of time dependence of carbon dioxide, methane, nitrous oxide, and chlorofluorocarbons and of radiation dependence on levels of these gases are given by Mitchell in [12] and by the Carbon Dioxide Information Analysis center in [1].

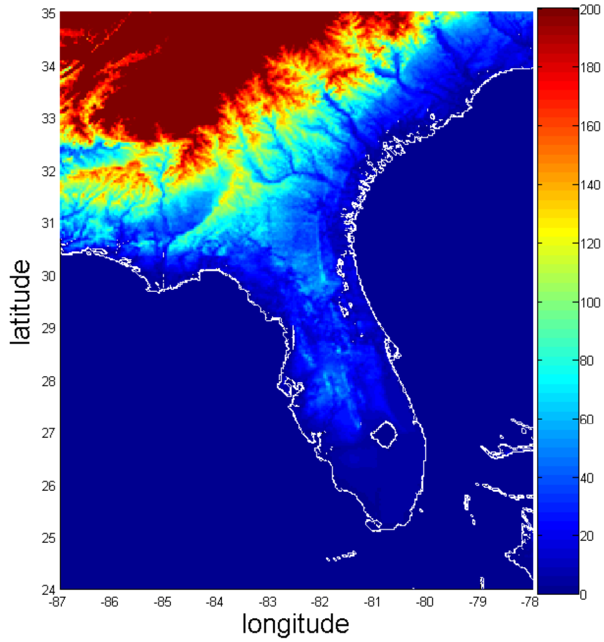

Figure 6: A plot of the topography of Florida.

6 Results

6.1 Rising temperatures

To test whether our model produces reasonable predictions, we numerically integrated equation (4.4) from 1971 to 2007. See Figure 7 for the results of this simulation plotting along with known numerical data. The agreement is quite good, and in both cases mean global temperature grows roughly linearly.

We numerically integrated equation (4.4) from 1992 to 2057; see Figure 8 for a color-coded plot of the predicted world temperature distribution in 2057. Note that in contrast to the measured 1990 distribution shown in Figure 4, the predicted 2057 distribution is nearly independent of longitude, except for at Greenland, on which the melting snow skews temperatures downward. See Figure 9 for a plot of predicted global mean temperature over the next fifty years, along with predicted mean temperature assuming emissions are cut by 25%, 50%, 75%, or 100%.

6.1.1 Sidenote: extremely long-term predictions

On timescales of 50 years or so, our model has produced roughly linear results for both mean temperature and mean sea-level rise versus time. However, this linearity is not preserved on a 500-year timescale. See Figures 10 and 11 to see this.

These results show that our model predicts that mean temperature is eventually sublinear and sea-level is eventually superlinear on long timescales. It is worth noting that the predicted

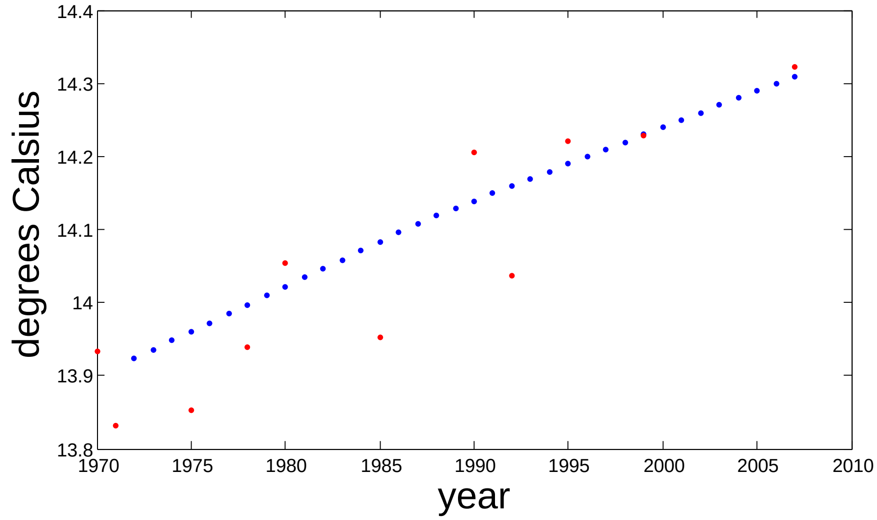

Figure 7: A plot of empirical measurements of mean global temperature versus time, along with predictions from our model. Red points are empirical; blue are predicted.

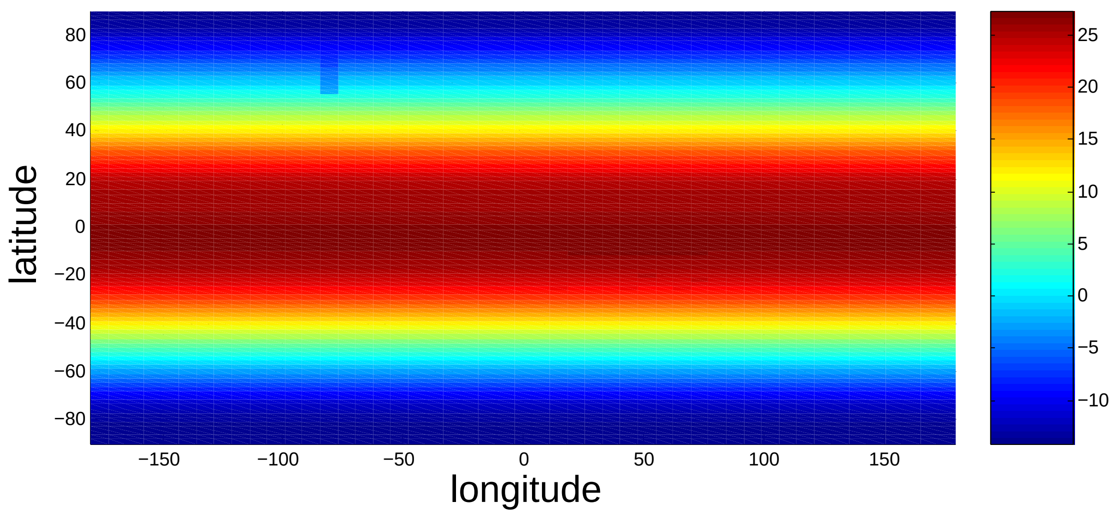

Figure 8: 2057 Earth temperatures, as predicted by our model.

superlinearity of sea-level was predicted by Rahmstorf *et al.* in [18].

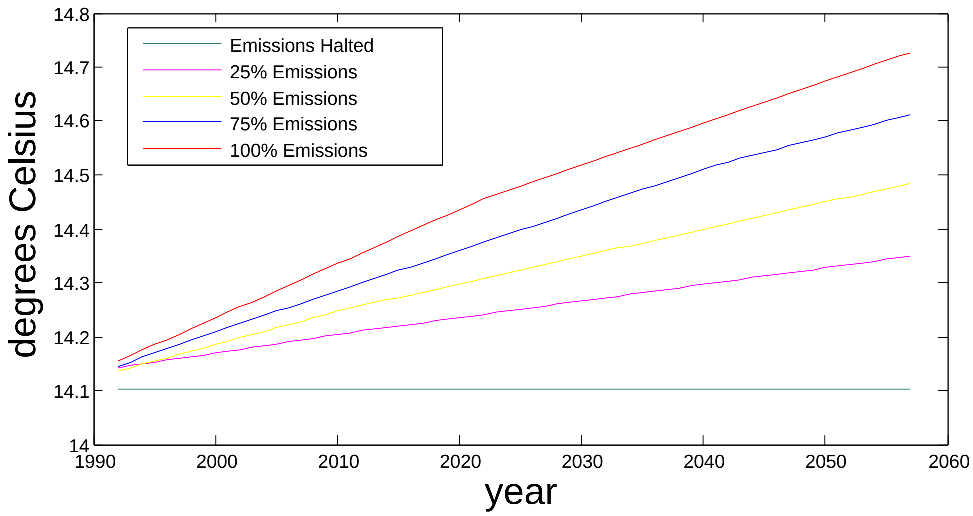

Figure 9: Predicted mean global temperature versus time, for a variety of greenhouse gas concentrations.

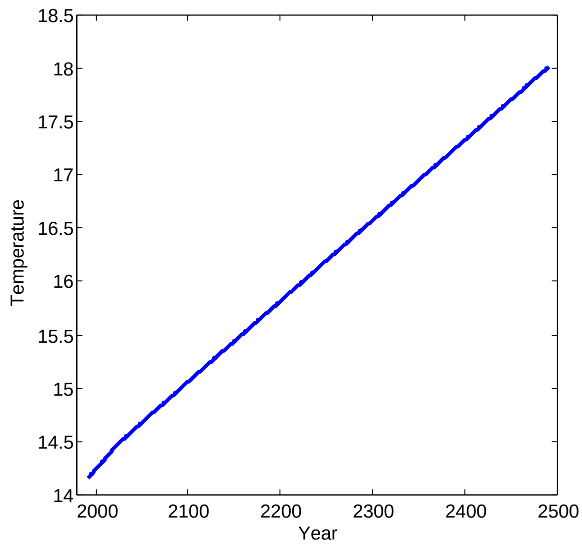

Figure 10: Predicted mean temperature versus time.

6.2 Rising sea levels

Our model can also predict variation in sea level. See Figure 12 for a plot of total predicted sea level increase versus time, plotted in each of the cases of assuming our current level of greenhouse gas emissions or assuming a 25%, 50%, 75%, or 100% reduction in emissions. It is important to

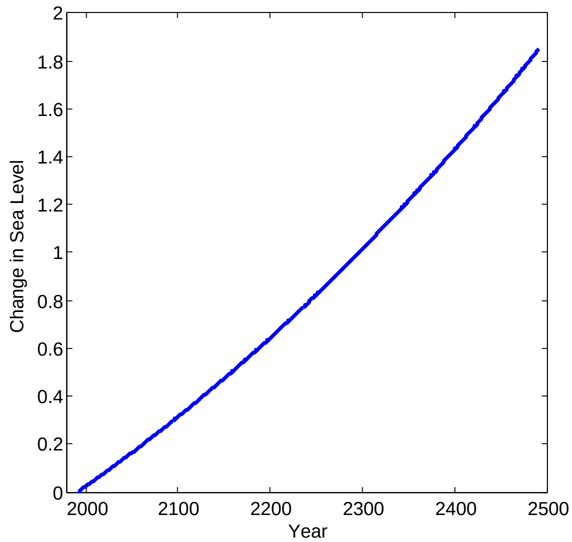

Figure 11: Predicted average sea level versus time.

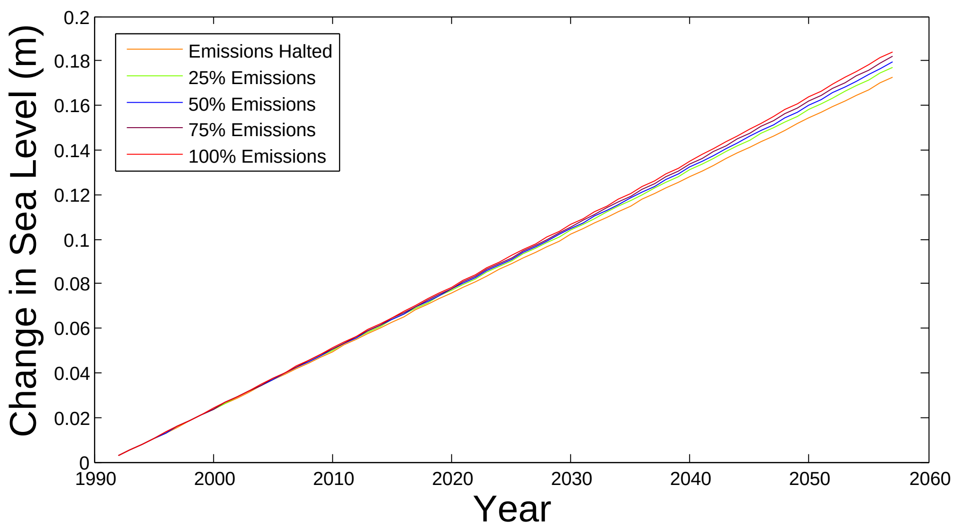

Figure 12: Predicted sea levels versus time, for a variety of greenhouse gas concentrations.

note that

- Reducing greenhouse gas emissions will not significantly affect sea-level rise.

6.3 Dropping salinity levels

Changes in global temperature appear to significantly alter the salinity of the Earth's oceans. Rising temperatures in the Northern hemisphere have caused melting of polar ice and increased precipitation, introducing additional fresh water to the ocean and lowering salinity [17]. Current studies have found that the salinity of waters in the subtropical Atlantic Ocean regions has progressively increased, while the salinity of the water close to the North and South poles has dropped [2]. These changing patterns in salinity have the effect of slowing global ocean currents [17].

Ocean currents are continuous streams of water that flow due mainly to differences in temperature and salinity, between two regions on the globe. These currents play a critical part in determining the climate around the world. In particular, the Gulf Stream, which runs from the Atlantic Ocean to Northern Europe, plays a significant role in moderating the climate of Florida [2]. Current models predict that if greenhouse gases rise and salinity continues to fall at its current rate, the Gulf Stream could be significantly slowed in 100 years, and the climates would face much colder temperatures and harsher winters. Our model predicts an increase in temperature roughly linear with current trends, and therefore that salinity will be a significant issue in the future.

7 Effect of global warming on the state of Florida

7.1 Previously conjectured effects

Global warming, and in particular the rising of the sea level and increasing ocean temperature, will pose a serious challenge to Florida in coming decades [6]. It is predicted that global warming will negatively affect a number of areas:

- Human health

  Higher average temperatures would have a number of effects on human health. Firstly, it would increase the number number of cases of heat stroke significantly. In Tampa alone, around 28 people die every year from heat related deaths. Experts predict that this number could easily double with even a 1.1°C rise in average temperature [3]. Secondly, warmer temperatures provide better conditions for disease-carrying insects, which would increase the transmission of some dangerous diseases [6]. Lastly, increased temperatures and greenhouse gas emissions would significantly worsen air quality [6].

- Economic prosperity

  Economically, global warming would pose a few challenges to Florida. Agriculture in Florida is largely affected by the supply of water, and global warming would likely decrease the soil moisture and result in the need for more irrigation for farmers. This would decrease crop yields and significantly hurt local economies [3]. Also, flooding of coastal cities and areas of large population would harm tourism in some areas [6].

- The environment and local ecosystems

  Ecologically, the large number of forests in Florida would be adversely affected by the rising temperatures. Typically, these ecosystems thrive only in specific temperatures and environments, and even small changes in the climate of Florida would significantly alter the growth dynamics of these areas. Additionally, many of the coastal marshlands and swamps would be effectively destroyed by saltwater due to an increased sea level.

Clearly, understanding global warming is important if we care about the well-being of Florida. Already, scientists and researchers have observed changes that coincide with the effects of the predicted climate changes [6]. In the past century, the average temperature in Florida has risen around 0.7°C, and the 9 warmest years of the past century have all occurred in the last 14 years [3]. Other recent problems include eroding shorelines, an increased risk of forest fires, increased salt levels in freshwater aquifers, and warmer land and sea temperatures [6].

7.2 Predicted effects from model

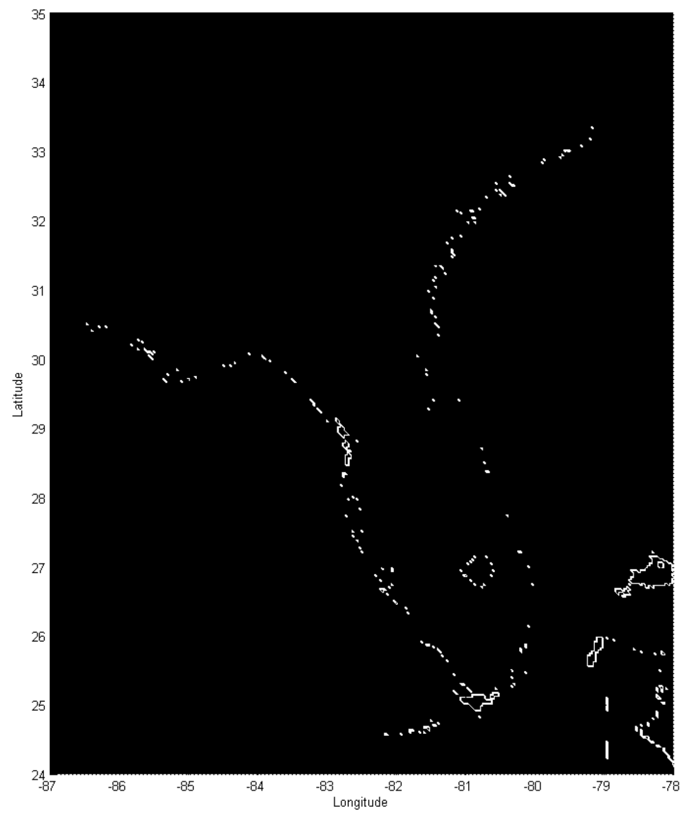

Figure 13: A plot of portion of Florida that will be underwater in 2057, as predicted by our model.

See Figure 13 for a plot of the portions of Florida that will be underwater in 2057, as predicted by our model. Unfortunately, the cartographical projection used for the population data is not the same as the projection used for the topographical data, and because of this we cannot determine exact population displacement.

8 Conclusions

We conclude that reducing greenhouse gas emissions will have no significant effect in curbing the global rise of sea level, and therefore will not affect number of Floridians displaced in the next fifty years.

Our model indicates that sea level increases linearly over time and superlinearly for timescales on the order of 500 years. Varying the global rate of CO2 emissions will have very little effect on changing sea levels in the next few decades. Even by reducing the current global output rate of greenhouse gases by the optimistic value of 50

- The sea level will rise significantly over the next half century, with or without a drastic change in greenhouse gas emissions.

Based on the predicted sea level increase, some major cities in Florida, including Miami and Tampa, would be faced with significant flooding. As such, while curbing greenhouse gas emissions would be beneficial in controlling air quality and temperature rise, it is not an effective preventive measure for stopping sea level rise. Thus, it is not effective in saving large portions of the Floridian population from being displaced. We suggest that the State of Florida look into measures that combat the effects of sea level rise, rather than trying to control it. Current proposed measures include:

- Constructing sea walls near large population areas
- Protecting freshwater aquifers from seawater
- Adopting measures to save valuable natural habitats

All of these plans should be taken into consideration in order to prevent considerable damage to Florida's cities, coastal environment, economy and infrastructure.

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